FYS113 - KOLLOKVIE OPPGAVER

1. Stråling - fotoner - Bohrs atommodell

Oppgave 1.1 Typiske energier

a) Vi skulle vise at $1eV = 1{,}602 \cdot 10^{-19} J$.

$\Rightarrow$ Ladningen til elektronet er $1{,}602 \cdot 10^{-19} C$ og spenningen V kan skrives som $V = J/C$.
Slik at $1volt \cdot q \Rightarrow 1volt \cdot e \Rightarrow 1{,}602 \cdot 10^{-19} C \cdot 1 J/C = \underline{1{,}602 \cdot 10^{-19} J}$

b) Energi økningen for følgende spenninger 10V, 50kV og 1MV.
Det gir oss følgende.

$\Delta E_k = 1{,}602 \cdot 10^{-19} C \cdot 10 J/C = \underline{1{,}602 \cdot 10^{-18} J}$

$\Delta E_k = 1{,}602 \cdot 10^{-19} C \cdot 10^{4} J/C = \underline{8{,}01 \cdot 10^{-15} J}$

$\Delta E_k = 1{,}602 \cdot 10^{-19} C \cdot 10^{5} J/C = \underline{1{,}602 \cdot 10^{-13} J}$

Det er bedre å sette opp en tabell når vi skal regne ut det oppgaven spør etter. Men først skal vi utlede hastigheten fra relativistiske ligninger.

$\Downarrow$

$$K = \frac{mc^2}{\sqrt{1-(v/c)^2}} - mc^2 \iff \frac{K}{mc^2} = \frac{1}{\sqrt{1-(v/c)^2}} - 1$$

$$\frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1-(v/c)^2}} \iff \left(\frac{K}{mc^2} + 1\right)^2 = \frac{1}{1-(v/c)^2}$$

$$\left(\frac{K}{mc^2} + 1\right)^2 = \frac{c^2}{c^2 - v^2} \iff \left(\frac{K}{mc^2} + 1\right)^2 = \frac{c^2}{c^2 - v^2}$$

$$\frac{c^2 - v^2}{c^2} = \frac{1}{\beta} \iff 1 - \left(\frac{v}{c}\right)^2 = \frac{1}{\beta}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{\beta} \iff v = \sqrt{c^2\left(1 - \frac{1}{\beta}\right)} = c\sqrt{1 - \frac{1}{\left(\frac{K}{mc^2} + 1\right)^2}}$$

Slik at tabellen settes nå slik:

U	E_k/J	E_k/eV	$v_1 = \sqrt{\frac{2E_k}{m}}$	$v_2 = c\sqrt{1 - \frac{1}{(1+\frac{E_k}{E_0})^2}}$	E_k/E_0
10V	$1{,}6\cdot10^{-18}$ J	10eV	$1{,}87\cdot10^{6}$ m/s	$1{,}87\cdot10^{6}$ m/s	$2\cdot10^{-5}$
$5\cdot10^{4}$V	$8\cdot10^{-15}$ J	50keV	$1{,}326\cdot10^{8}$ m/s	$1{,}24\cdot10^{8}$ m/s	0,1
$1\cdot10^{6}$V	$1{,}0\cdot10^{-13}$ J	1 MeV	$5{,}9\cdot10^{8}$ m/s	$2{,}8\cdot10^{8}$ m/s	2

E_k/E_0 skal uttrykke: Feilen man gjør i prosentandel, så disse faktorene skal ganges med 100%, se særlig nederst, hvor vi gjør feil på 200%!!

b)

Vi skal beregne den elektriske potensialet for to ladninger med $q = e$, med avstand $r = 10^{-10}$ m.

e ← $r = 10^{-10}$ m → e

$$E_p = \frac{e^2}{4\varepsilon_0 \pi r} = 14{,}4\,(1{,}602\cdot10^{-19}\,C\cdot V) = \underline{14{,}4\,eV}$$ ja det er naturlig

c)

Vi skal sammenligne størrelsesorden mellom gravitasjonell potensial og med elektrisk potensial.

$$E_p^G = -\gamma\frac{Mm}{r} = -1{,}0\cdot10^{-57}\,J \quad \text{og} \quad E_p^C = -\frac{e^2}{4\pi\varepsilon_0 r} = -2{,}25\cdot10^{-18}\,J$$

Det gir meg da: $\frac{E_p^G}{E_p^C} = \underline{2{,}3\cdot10^{39}}$

Det betyr at vi skal se bort fra gravitasjon's potensialet.

d) Hvile energien er gitt ved $E_0 = m_0c^2$

elektron: $E_0^e = m_e c^2 = 8{,}2\cdot10^{-14}\,J = 511\,keV$

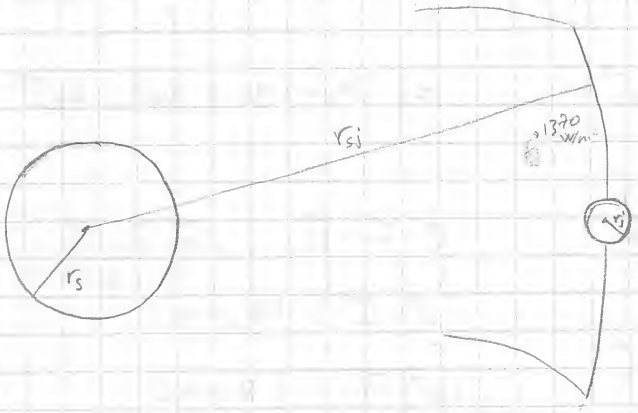
proton: $E_0^p = m_p c^2 = 1{,}5\cdot10^{-10}\,J = 938\,MeV$

1,2 Sort stråling

a) Vi skal beregne temperaturen til sola. Vi har $r_S = 6{,}96 \cdot 10^8$ m og $r_{sj} = 6378$ km.

jorden mottar strålingen som er på 1370 W/m², på en avstand $1{,}5 \cdot 10^{11}$ km.

$R(I) = \sigma T^4$, $\sigma = 5{,}67 \cdot 10^{-8}$ W/m²K⁴



stråling/m²s på sola, $\Rightarrow E_s$, stråling/m²s på sol-jord avstand $\Rightarrow E_j$

slik at $E_s = E_j$ må være slik.

totaleffekt på sola: $E_s = W_s \cdot 4\pi r_S^2$, totaleffekt i avstand r_{sj} : $E_j = W_{sj} 4\pi (r_{sj})^2$

$E_s = E_j \Rightarrow$ $W_s 4\pi r_S^2 = W_{sj} 4\pi (r_{sj})^2$

$$W_s = \frac{4\pi (r_{sj})^2 \cdot W_{sj}}{4\pi (r_S)^2}$$

$$W_s = \frac{(r_{sj})^2}{(r_S)^2} W_{sj} = \frac{(1{,}5 \cdot 10^{11})^2}{(6{,}96 \cdot 10^8)^2} 1370 = 6{,}36 \cdot 10^7 \text{ W}$$

slik at Stefan-Boltmanns lov kan brukes:

$$W_s = \sigma T^4 \Leftrightarrow T = \sqrt[4]{\frac{W_s}{\sigma}} = 5787{,}95 \text{ K} = \underline{5787 \text{ K}}$$

b) Beregne energiabsorbsjon på jorda når atmosfæren reflekterer bort 30% av innkommende lys.

jordkloden ser som ut pannekake, så stor avstand, derfor (πr_j^2).

$$\Rightarrow 0{,}7 \cdot 1370 \frac{\text{W}}{\text{m}^2} (\pi r_j^2) \approx \underline{1{,}22 \cdot 10^{17} \text{ W}}$$

c) termisk likevekt: total stråling inn = total stråling ut.

$$\pi (r_j)^2 \cdot 959{,}0 \text{ W/m}^2 = 4\pi (r_j)^2 \cdot R_{ut}$$

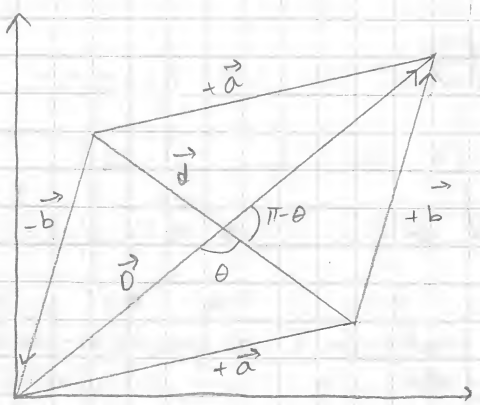
$$R_{ut} = 3{,}91 \cdot 10^{30} \text{ W/m}^2$$

slik at: $T\sigma = R_{ut} \Leftrightarrow T = \sqrt[4]{\frac{R_{ut}}{\sigma}} = \underline{254 \text{ K}} \Leftrightarrow \underline{-19°\text{C}}$

1.3 Komplekse tall

Vi skal vise at $|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2|z_1|^2 + 2|z_2|^2$, hvor z_1 og z_2 er komplekse tall.

$z_1 = x_1 + iy_1 \equiv a_x\vec{i} + a_y\vec{j}$, $z_2 = x_2 + iy_2 \equiv b_x\vec{i} + b_y\vec{j}$



$\vec{a} + \vec{b} = \vec{D}$, $\vec{a} - \vec{b} = \vec{d}$

$|\vec{a} + \vec{b}| = |z_1 + z_2|$

$|\vec{a} - \vec{b}| = |z_1 - z_2|$

+1 $\left(\frac{D}{2}\right)^2 + \left(\frac{d}{2}\right)^2 - 2\,\frac{D}{2}\,\frac{d}{2}\cos\theta = a^2$

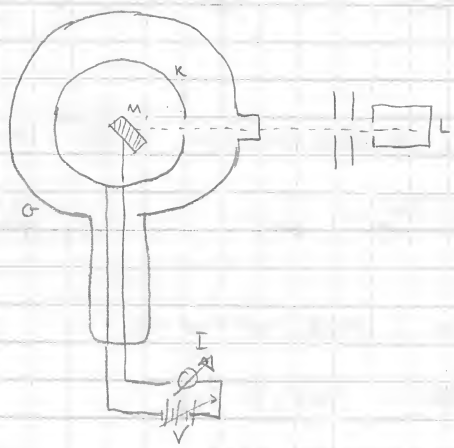
+1 $\left(\frac{D}{2}\right)^2 + \left(\frac{d}{2}\right)^2 - 2\,\frac{D}{2}\,\frac{d}{2}\cos(\pi-\theta) = b^2$

(+1, +1) → $2\left(\frac{D}{2}\right)^2 + 2\left(\frac{d}{2}\right)^2 = a^2 + b^2$

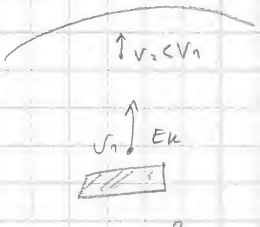
$|D|^2 + |d|^2 = 2a^2 + 2b^2$

$$\underline{|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2}$$

1.4 Fotoelektrisk effekt



a) Det den viser er at vi får strøm når vi har negativ spenning. Det skyldes at fotonene greier å få slått ut elektroner og får med seg noen kinetisk energi gitt ved $E_{potensial} = eV$. Siden potensialet er negativ, så er feltretningen fra M til K. Siden eV var ikke tilstrekkelig nok til å komme opp til K, slik at den falt først tilbake til M p.g.a. feltretningen. Slik at vi får strøm. Vi øker potensialet og går igjen til å øke kinetiske energien til elektronet, det fører til at flere og flere nå kan nå til K. Etter hvert får vi rettlinjet p.g.a. at den bølgelengden kan bare skyte ut en maks elektroner, uansett vi øker potensialet, så får vi fortsatt den samme mengde frigjorte elektroner slik at vi får konstant strøm.



b) Det å fordoble lysintensiteten fører til at vi får doblet strømmen, for vi har fått nå dobbelt så mange fotoner.

c) Vi har gitt at $hf = 4{,}81\,eV$, vi bruker formelen: $hf = W + eV_0$

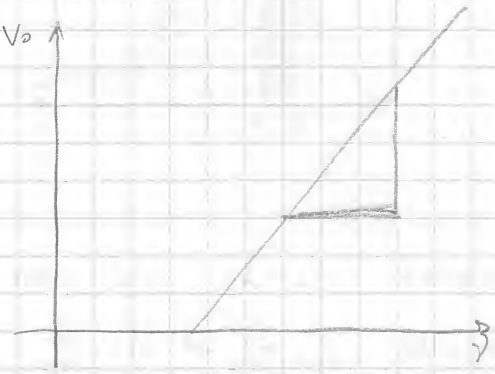
altså $W = hf - eV_0 \Leftrightarrow W = 4{,}81\,eV - e(0{,}7)V = \underline{\underline{4{,}1\,eV}}$

d)

$\lambda_1 = 2{,}40 \cdot 10^{-7}$ $\quad E_{\lambda_1} = \frac{c}{\lambda_1} \cdot h = \underline{5{,}17eV}$ $\quad K_{max} = 1{,}1eV \rightarrow \underline{V_0 = -1{,}1V}$

$\lambda_2 = 3{,}50 \cdot 10^{-7}$ $\quad E_{\lambda_2} = \frac{c}{\lambda_2} h = \underline{3{,}54eV}$ $\quad K_{max} = 0$ fjern plate

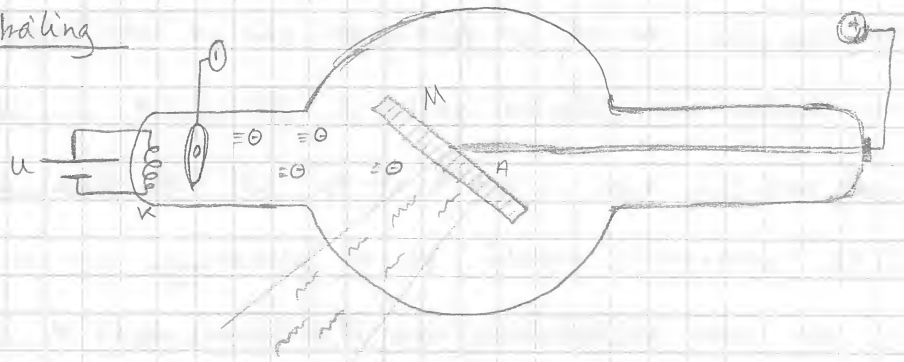
e)



$hf = W_1 + qU_T$

$U_T = \frac{h}{q} f - \frac{W_1}{q}$ $\quad \frac{h}{q}$ bestemmer stigningen

1.5. Røntgenstråling



a)

Selve røret er luftfritt. Elektroner med ladning -e frigjøres fra glødende katode K med glødespenning U. Elektronene akselereres mot anoden A ved hjelp av akselerasjonsspenningen U. Like før elektronene treffer anoden, har de fått hastighet v. Energiprinsippet gir da $\frac{1}{2}mv^2 = Ue$.
Når elektronene treffer wolfram overflaten, gir de fra seg en del av sin energi i form av elektromagnetisk stråling. Det er denne strålingen vi kaller røntgen stråling.
Denne røntgenstrålingen kaller vi ofte for bremsestråling. Bremsestrålingen fra et røntgenrør har et kontinuerlig spektrum, ettersom elektronenes akselerasjon varierer kontinuerlig under oppbremsningen. → Men røntgenrøret sender også ut en annen type stråling. Denne strålingen sendes ut av atomene i anodematerialet når de innkommende elektronene støter mot atomene i anodematerialet. Denne røntgenstrålingen kaller vi karakteristiske stråling, fordi den er karakteristisk for det grunnstoffet anoden er laget av. Den karakteristiske strålingen har et linjespektrum.

b)

$\lambda\nu = c$

Antall fotoner per sekund i intervallet $\lambda_1 \to \lambda_2$ må være lik antall fotoner per sekund i intervallet $\nu_2 = \frac{c}{\lambda_2} \to \nu_1 = \frac{c}{\lambda_1}$, $\underline{\lambda_1 < \lambda_2}$ } #

$$\int_{\lambda_1}^{\lambda_2} I_\lambda(\lambda)\,d\lambda = \qquad \lambda = \frac{c}{\nu} \xrightarrow{\text{Derivere:}} d\lambda = \frac{-c}{\nu^2}\,d\nu \qquad \nu_1 = \frac{c}{\lambda_1} > \nu_2 = \frac{c}{\lambda_2}$$

$$= \int_{\nu_1}^{\nu_2} I_\lambda\left(\frac{c}{\nu}\right)\left(\frac{-c}{\nu^2}\right)d\nu = \int_{\nu_2}^{\nu_1} \frac{c}{\nu^2} I_{(\lambda)}\left(\frac{c}{\nu}\right) \overset{I}{=} \int_{\nu_2}^{\nu_1} I_\nu(\nu)\,d\nu \qquad \text{J.F.R. 9.6}$$

$$\Rightarrow \underline{I_\nu(\nu) = \frac{c}{\nu^2} I_\lambda\left(\frac{c}{\nu}\right)} \qquad \left(\text{tilsvarende: } I_\lambda(\lambda) = \frac{c}{\lambda^2} I_\nu\left(\frac{c}{\lambda}\right)\right)$$

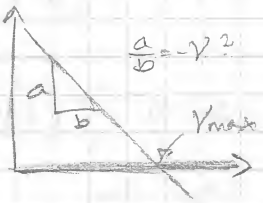
Y. $$\begin{cases} I(\lambda) = \dfrac{\partial I(\lambda)}{\partial \lambda} = \dfrac{\partial I_{tot}}{\partial \nu}\dfrac{\partial \nu}{\partial \lambda} = I(\nu)\dfrac{\partial \nu}{\partial \lambda} = \underline{I(\nu)\cdot\frac{c}{\lambda^2}} \\ I(\nu) = \dfrac{\partial I_{tot}}{\partial \nu} \end{cases}$$

J. $I(\lambda)\,d\lambda = I(\nu)\,d\nu \rightarrow$ transportere fra ett intervall til den andre intervalle.

$$\underline{I(\nu) = I(\lambda)\left|\frac{d\lambda}{d\nu}\right|}$$

c) λ_{MIN}, ν_{MAX} } All energi fra potensialet til ett foton: $\underline{V\cdot e = \frac{hc}{\lambda_{min}} = h\nu_{max}}$

d) $I_\nu = k(\nu_{max} - \nu)$ fra figur 2.b. linje 2.



f) Vi skal integrere opp $\int I(\nu)\,d\nu$, siden ci har en trekant, så dette er greit. Hvis ikke så,

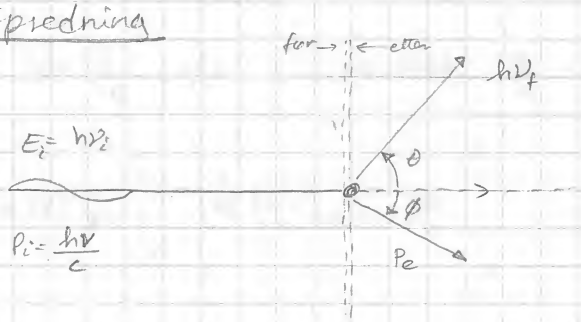
$$I = \int_0^{\nu_{max}} I_\nu\,d\nu = \int_0^{\nu_{max}} k(\nu_{max} - \nu) = \left|-\frac{1}{2}k(\nu_{max} - \nu)^2\right|_0^{\nu_{max}} = \underline{\frac{1}{2}k\nu_{max}^2}$$

$$\nu_{max} = V\cdot\frac{e}{h} \Rightarrow \underline{I = \frac{1}{2}k\left(V\cdot\frac{e}{h}\right)^2} \Rightarrow \underline{I \sim V^2}$$

g) Elektroner slås løs fra K-skallet (eventuelt de andre skallene), Elektronene fra L-skallet eller lengre ut faller inn og foton med bestemt bølgelengde sendes ut.

1.6 Compton spredning

a)



De to prinsippene er 1) bevaring av energi og 2) bevaring av bevegelsesmengde.

Utlede Comptons formel:

1): $h\nu_i + m_e c^2 = h\nu_f + \sqrt{(m_e c^2)^2 + (p_e c)^2}$

2): $\vec{p_i} = \vec{p_f} + \vec{p_e}$

2: $\vec{p_i} - \vec{p_f} = \vec{p_e} \Rightarrow$ KVADRERE $\Rightarrow (\vec{p_i} - \vec{p_f})^2 = (\vec{p_e})^2$

$$\vec{p_i}^2 + \vec{p_f}^2 - 2\vec{p_i}\vec{p_f}\cos\theta = \vec{p_e}^2 \quad / \cdot c^2$$

$$\vec{p_i}^2 c^2 + \vec{p_f}^2 c^2 - 2\vec{p_i}\vec{p_f} c^2 \cos\theta = \vec{p_e}^2 c^2$$

1): $h\nu_i + m_e c^2 - h\nu_f = \sqrt{(m_e c^2)^2 + (p_e c)^2} \Rightarrow$ KVADRERE

$$(h\nu_i + m_e c^2 - h\nu_f)^2 = (m_e c^2)^2 + (p_e c)^2 \Leftrightarrow (h\nu_i + m_e c^2 - h\nu_f)^2 - m_e^2 c^4 = p_e^2 c^2$$

2 = 1'): $(h\nu_i)^2 + (h\nu_f)^2 - 2h\nu_i h\nu_f \cos\theta = (h\nu_i + mc^2 - h\nu_f)^2 - (m_e c^2)^2$

Høyre siden:

$$(h\nu_i + mc^2 - h\nu_f)^2 - (m_e c^2)^2 = (h\nu_i)^2 + (m_e c^2)^2 + (h\nu_f)^2 + 2(m_e c^2)(h\nu_i) - 2(h\nu_i)(h\nu_f) - 2(mc^2)(h\nu_f) - (m_e c^2)^2$$

Slik at:

$$(h\nu_i)^2 + (h\nu_f)^2 - 2h\nu_i h\nu_f \cos\theta = (h\nu_i)^2 + (h\nu_f)^2 + 2(m_e c^2)(h\nu_i) - 2(h\nu_i)(h\nu_f) - 2(mc^2)(h\nu_f)$$

$$-(h\nu_i)(h\nu_f)\cos\theta = (m_e c^2)(h\nu_i) - (h\nu_i)(h\nu_f) - (mc^2)(h\nu_f)$$

$$-(h\nu_i)(h\nu_f)\cos\theta + (h\nu_i)(h\nu_f) = m_e c^2 (h\nu_i - h\nu_f)$$

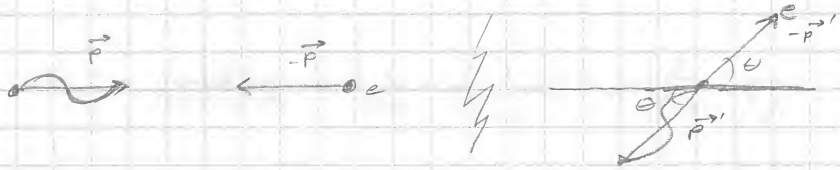
$$h\nu_i h\nu_f (1 - \cos\theta) = m_e c^2 (h\nu_i - h\nu_f) \quad : h^2 c^2$$

$$\frac{\nu_i}{c}\frac{\nu_f}{c}(1 - \cos\theta) = \frac{m_e c}{h}\left(\frac{\nu_i}{c} - \frac{\nu_f}{c}\right), \quad c = \nu\lambda$$

$$\frac{m_e c}{h}\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right) = \frac{(1 - \cos\theta)}{\lambda_i \lambda_f} \Leftrightarrow \underline{\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)}$$

b) Ved støtet avgir fotonet noe energi til elektronet, altså har nå mindre energi $E_f > E_i$ → Da må fotonet ha lengre bølgelengde → $\underline{\underline{E = h\nu}}$

c)



Energien: $pc + \frac{p^2}{2m} = p'c + \frac{p'^2}{2m} \Leftrightarrow \underline{\underline{p' = p}}$ bølgelengden er uendret

d) Er dette situasjon mulig:

$$h\nu_i + m_0c^2 = h\nu_f + \sqrt{m_e^2c^4 + (p_ec)^2}$$

$$(h\nu_i + m_0c^2 - h\nu_f)^2 = \left(\sqrt{m_e^2c^4 + (h\nu)^2}\right)^2$$

$$(h\nu_i)^2 + (m_0c^2)^2 + \overset{\to 0}{(h\nu_f)^2} + 2(m_0c^2)(h\nu_i) - 2\overset{\to 0}{(h\nu_i)(h\nu_f)} - 2\overset{\to 0}{(m_0c^2)(h\nu_f)} = m_e^2c^4 + (h\nu_i)^2$$

$$\cancel{(h\nu_i)^2} + \cancel{(m_0^2c^4)} + 2h\nu_i m_0c^2 = \cancel{m_e^2c^4} + \cancel{(h\nu_i)^2}$$

$2h\nu_i m_0c^2 = 0 \Leftrightarrow$ UMULIG, for betingelsene var $\nu_i \neq 0$ og $m_0 \neq 0$.

1.8 Bohrs atommodell

Synlig lys har bølgelengde i området 4000 - 7000 Å (400nm - 700nm)

$Å = 10^{-10}m = 1{,}0 \cdot 10^{-9}$

a) Bruker da formelen: $\frac{1}{\lambda} = \frac{\nu}{c} = \frac{E_0}{hc}\left|\frac{1}{n_f^2} - \frac{1}{n_i^2}\right| = R_\infty\left|\frac{1}{n_i^2} - \frac{1}{n_f^2}\right|$, $R = 1{,}097 \times 10^7$

Slik at $\frac{1}{\lambda_{max}} = R_\infty\left|\frac{1}{1^2} - \frac{1}{2^2}\right| \Rightarrow 1{,}22 \cdot 10^{-7}m \Rightarrow \underline{122\,nm}$

$$\frac{1}{\lambda_{min}} = R_\infty\left|\frac{1}{1^2} - \frac{1}{\infty}\right| \Rightarrow \underline{\underline{91{,}2\,nm}}$$

Begge er utenfor den synlige områder ved alle overganger.

b) Tilsvarende for her

$$\frac{1}{\lambda_{max}} = R_\infty \left| \frac{1}{n_f^2} - \frac{1}{n_i^2} \right| = 1{,}097 \cdot 10^7 \left| \frac{1}{2^2} - \frac{1}{3^2} \right| \Rightarrow \lambda_{max} = 6{,}6 \cdot 10^{-7}\,m = \underline{656\,nm}$$

$$\frac{1}{\lambda_{min}} = R_\infty \left| \frac{1}{n_f^2} - \frac{1}{n_i^2} \right| = R_\infty \left| \frac{1}{2^2} - \frac{1}{\infty} \right| \Rightarrow \underline{365\,nm}$$

c)

$$\frac{1}{\lambda_{max}} = R_\infty \left| \frac{1}{4} - \frac{1}{n} \right| \Leftrightarrow \frac{1}{\lambda_m R_\infty} - \frac{1}{4} = -\frac{1}{n_i^2}$$

$$\frac{1}{n_i} = \sqrt{\frac{1}{4} - \frac{1}{\lambda R_\infty}}$$

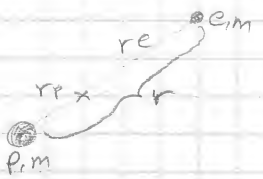
$\lambda = 700\,nm \Rightarrow \quad n = 2{,}88 \approx 3$

$\lambda = 400\,nm \Rightarrow \quad n = 6{,}72 \approx 7$

$\Rightarrow$ Det betyr at vi får $n = 3$, $n = 4$, $n = 5$, $n = 6$, $n = 7$

Det blir altså rundt 5 spektrallinjer i det synlige området.

1.9. Bohrs modell



a) $r = r_p + r_e$

Banespinnet: $L_{tot} = L_e + L_p = m_e \omega r_e^2 + m_p \omega r_p^2 = n\hbar$

$= m_e \omega (r - r_p)^2 + m_p \omega r_p^2 = n\hbar$

1) $r_e + r_p = r$

2) $r_e m_e = r_p m_p$

$= \frac{m_e m_p}{m_e + m_p} \omega r^2 = \mu \omega r^2$

b) Tar utgangspunkt i Bohr utregningen erstatter m_e med μ

$$E_{tot} = E_k + E_p = \frac{1}{2} m v^2 + \frac{1}{2} \mu v^2 - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\frac{mv^2}{y} = \frac{e^2}{4\pi\epsilon_0 r^2} \cdot \left| \frac{y}{2} \right.$$

$$\frac{\mu r^2}{x} = \frac{e^2}{4\pi\epsilon_0 r^2} \left| \cdot \frac{x}{2} \right.$$

$$\frac{1}{2}mv^2 + \frac{1}{2}Mv^2 = \frac{e^2}{4\pi\varepsilon_0 r^2}\left(\frac{x}{2} + \frac{y}{2}\right) \qquad \begin{matrix} x = r_e \\ y = r_p \end{matrix}$$

$$E_{tot} = -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r}$$

$$\frac{mv^2}{x} = \frac{e^2}{4\pi\varepsilon_0 r^2} = m\frac{x^2\omega^2}{x} \qquad \Big| \; x = r\frac{M}{m+M}$$

$$\omega^2 = \frac{e^2}{4\pi\varepsilon_0 r^3 \mu}$$

$$L = \mu\omega r^2 = n\hbar \qquad \omega^2 = \left(\frac{n\hbar}{\mu r^2}\right)^2 \qquad , r = \frac{4\pi\varepsilon_0}{e^2\mu}(n\hbar)^2$$

$$E_{tot} = -\frac{\mu}{2}\left(\frac{e^2}{4\pi\varepsilon_0 n\hbar}\right)^2 = \underline{\underline{-\frac{\mu e^4}{8\varepsilon_0^2 h^2 n^2}}}$$

c)

1) $$E_\alpha(m_e) = h\nu_e = \frac{-m_e e^4}{8\varepsilon_0^2 h^2}\left|\frac{1}{2^2} - \frac{1}{3^2}\right|$$

2) $$E_\alpha(\mu) = h\nu = \frac{-\mu e^4}{8\varepsilon_0^2 h^2}\left|\frac{1}{2^2} - \frac{1}{3^2}\right|$$

$$\begin{matrix} 1) \\ 2) \end{matrix} \quad \frac{\nu_e}{\nu_\mu} = \frac{m}{\mu} = m\frac{M+m}{M\cdot m_e} = 1{,}00054464$$

$$\underline{\underline{\lambda_\mu = \lambda_m \cdot 1{,}00054464}}$$

d)

1) $$h\nu_\alpha(H) = \frac{\mu(H)\, e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \quad \text{mäting}$$

$$\lambda_\alpha H = 6561{,}01\,\text{Å}$$

2) $$h\nu_\alpha(D) = \frac{\mu(D)\cdot e^4}{8\cdot\varepsilon_0^2\cdot h^2}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_\alpha^0 = 6562{,}80\,\text{Å}$$

$$\frac{\nu_\alpha(H)}{\nu_\alpha(D)} = \frac{\mu(H)}{\mu(D)} = \frac{\frac{m_e M_p}{m_e + M_p}}{\frac{m_e M_D}{m_e + M_D}} = \frac{1 + \frac{m_e}{M_p}}{1 + \frac{m_e}{M_D}} = \frac{\lambda_\alpha(D)}{\lambda_\alpha(H)}$$

$$\underline{\underline{M_D \simeq 2M_p}}$$

1.10 Bohrs atommodell

a) Regner ut energien til korteste og lengste bølgelengde

$$E_{maks} = h\nu = h\frac{c}{\lambda} = \frac{6{,}63\cdot10^{-34}\,Js\cdot 3{,}0\cdot10^{8}\,m/s}{400\cdot10^{-9}\,m} = \underline{3{,}10\,eV}$$

$$E_{min} = h\nu = h\frac{c}{\lambda} = \frac{6{,}63\cdot10^{-34}\,Js\cdot 3{,}0\cdot10^{8}}{700\cdot10^{-9}\,m} = \underline{1{,}77\,eV}$$

Energien ligger i $\underline{1{,}77 \leq E \leq 3{,}10}$

b) Spektrallinjer som gir synlig lys - hydrogen (Balmer-serien) $Z=1$

$$E = Z^2 E_0 \left|\frac{1}{n_f^2} - \frac{1}{n_i^2}\right|$$

$$n_i = \sqrt{\left(\frac{1}{n_f^2} - \frac{E}{E_0}\right)^{-1}} = \sqrt{\left(\frac{1}{4} - \frac{3{,}10}{13{,}6}\right)^{-1}} = \underline{6}$$

$$n_i = \sqrt{\left(\frac{1}{4} - \frac{1{,}77}{13{,}6}\right)^{-1}} = 3$$

For helium ($Z=2$)

$$n_i = \sqrt{\left(\frac{1}{n_f} - \frac{E}{2E_0}\right)^{-1}} = \sqrt{\left(\frac{1}{16} - \frac{3{,}10}{4\cdot13{,}6}\right)^{-1}} = 13$$

$$n_i = \sqrt{\left(\frac{1}{16} - \frac{1{,}77}{4\cdot13{,}6}\right)^{-1}} = 6$$

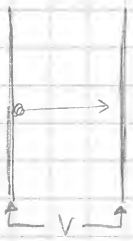
Spektrallinjer 8 stk.

c) $$h\nu = \frac{2\Delta E}{\sqrt{1+\frac{2\Delta E}{Mc^2}}+1}$$

$\lambda_{dB} = \lambda_C$

1.M. De Broglies Bølge

a) Vis at $\lambda = \frac{1,23}{\sqrt{V}}$ nm. (klassisk)



1) $\lambda = \frac{h}{p}$ 2) $\frac{1}{2}mv^2 = eV$ 3) $\frac{1}{2}mv^2 = \frac{p^2}{2m}$

slik at $\frac{1}{2}mv^2 = eV$

$$\frac{p^2}{2m} = eV$$

$$\frac{\frac{h^2}{\lambda^2}}{2m} = eV \quad \Leftrightarrow \quad \frac{h^2}{\lambda^2} = 2meV$$

$$\lambda^2 = \frac{h^2}{2meV}$$

$$\lambda = \sqrt{\frac{h^2}{2meV}} = \frac{h}{\sqrt{2meV}} = \frac{h}{\sqrt{2me}\sqrt{V}} = \underline{\underline{\frac{1,23\,nm}{\sqrt{V}}}}$$

b) relativistisk

1) $\lambda = \frac{h}{p}$ 2) $E_{tot} = \sqrt{(m_0c^2)^2 + (pc)^2}$ 3) $E_{tot} = eV + m_0c^2$

(2)^2 og (3)^2 $\Rightarrow$ 2) $(E_{tot})^2 = (m_0c^2)^2 + (pc)^2$

(3) $(E_{tot})^2 = (eV + m_0c^2)^2 = (eV)^2 + (m_0c^2)^2 + 2m_0c^2eV$

2=3: $(pc)^2 = (eV)^2 + 2m_0c^2eV$

$$p = \sqrt{\frac{(eV)^2 + 2m_0c^2eV}{c^2}} \quad (I)$$

(I) innsatt i 1): $\lambda = \frac{h}{p} = \frac{h}{\left(\frac{(eV)^2 + 2m_0c^2eV}{c^2}\right)^{1/2}} = \frac{h}{\left(\left(\frac{eV}{c}\right)^2 + 2m_0eV\right)^{1/2}}$

$$= \frac{h}{\left(2m_0eV + \frac{2m_0e^2V^2}{2m_0c^2}\right)^{1/2}} = \underline{\underline{\frac{h}{\sqrt{2m_0eV}\left(1 + \frac{eV}{2m_0c^2}\right)^{1/2}}}}$$

c)

1.12 Diffraksjon

a)

$\lambda = 3\text{Å}$

$\nu = \frac{c}{\lambda} = \underline{10^{18}\,\text{Hz}}$

b) $E = h\nu = \underline{4{,}14\,\text{keV}}$

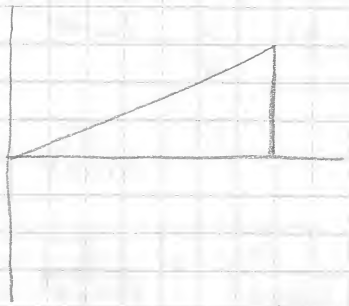
c) $\lambda = \frac{h}{p} = p = \frac{h}{\lambda}$

$E_k + m_0c^2 = \sqrt{(m_0c^2) + (pc)^2}$

$\underline{E_k = 16{,}8\,\text{eV}}$

d) elektronmikroskop billigere

1.13. Diffraksjon



$\frac{I_\theta}{I_m} = \frac{1}{2} = \frac{\sin\alpha x}{\alpha x}$ $\quad \alpha = 1{,}40$

$\alpha x = \frac{\pi a}{\lambda}\sin\theta$

$\frac{I}{I_\theta} = \left(\frac{\sin\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\frac{\pi a}{\lambda}\sin\theta}\right)^2$ $\qquad \left(\frac{\sin u}{u}\right)^2 = \frac{1}{2}$

$\frac{\sin u}{u} = \frac{1}{\sqrt{2}}$

$\underline{\sin u = \frac{u}{\sqrt{2}}}$

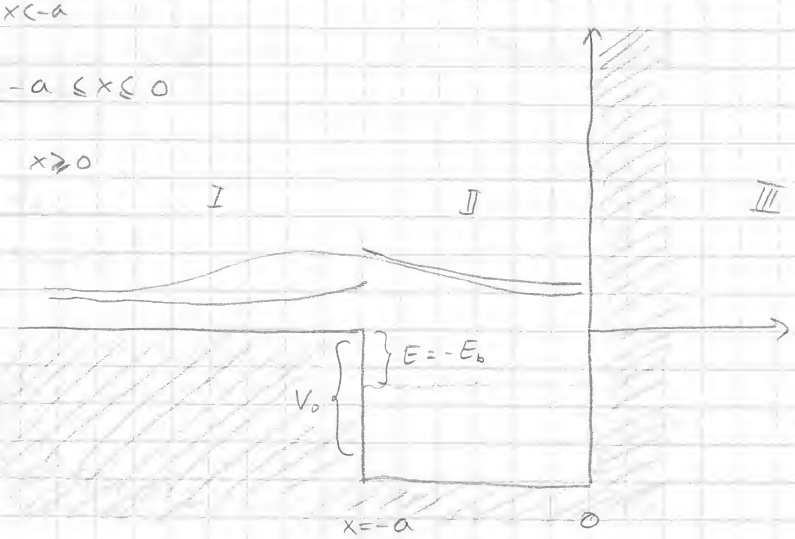
2.2

En partikkel med masse m og energi $E<0$ beveger seg langs x-aksen i potensialet

$V = 0$ for $x < -a$

$V = -V_0$ for $-a \leq x \leq 0$

$V = +\infty$ for $x \geq 0$



$$-\frac{\hbar^2}{2m}\psi_I'' + 0\cdot\psi_I = -E_b\psi_I$$

$$\psi_I'' = \frac{2mE_b}{\hbar^2}\psi_I = \alpha^2\psi_I$$

$$\psi_I = Ae^{\alpha x} + Be^{-\alpha x},$$

$\psi_I = C[e^{\alpha x} + e^{+\alpha x}]$ for det går mot taket, altså ikke normerbar på $-x$ aksen, derfor må vi ha + varianten.

Område I: $\psi_I(x) = Ce^{\alpha x}$

Område III: $\psi_{III}(x) = 0$

Område II: $-\frac{\hbar^2}{2m}\psi_{II}'' - V_0\psi_{II} = -E_b\psi_{II}$

$$\psi_{II}'' = \frac{2m}{\hbar^2}(E_b - V_0)\psi_{II}$$

$$\psi_{II}''(x) = -k^2\psi_{II}(x) \quad \text{÷ fortegn pga } E_b < V_0$$

$$\psi_{II}(x) = Ae^{ikx} + Be^{-ikx}$$

$\Rightarrow \psi_I(x=0) = A + B = 0 \rightarrow B = -A$

$$\psi_{II} = A\cos kx + iA\sin kx - A\cos kx + iA\sin kx$$

$$= 2Ai\sin kx$$

$\psi_I(-a) = \psi_{II}(-a)$ og $\psi_I'(-a) = \psi_{II}'(-a)$

$\alpha Ce^{\alpha x} = k2iA\cos(kx)$

$A + B = C$ alle størrelser må være endelige og at ψ er funnet så kan normaliseres. Det ville vært meningsløs om A eller B var uendelig mens C var endelig. Altså må A og B være deriverbar for at vi skal få endelige verdier, derfor kan ikke partikkelen oppføre seg slik som i figur.

1) $Ce^{-\alpha a} = 2iA\sin(\pm ka)$ 2) $\alpha Ce^{-\alpha a} = k2iA\cos(+kA)$

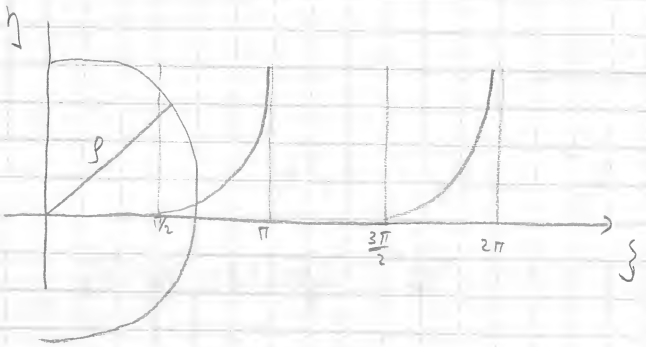
$\frac{2)}{1)} = \alpha a = -(ka)\cot g(ka)$

$\equiv \eta = -\xi\cot g(\xi)$ fra schrödingers ligning.

$\xi^2 + \eta^2 = \rho^2$

$= (ka)^2 + (\alpha a)^2 = a^2\frac{2m}{\hbar^2}(V_0 - E_b) + \frac{a^2 2m}{\hbar^2}E_b = \frac{2mV_0a^2}{\hbar^2} = \rho^2$

For å få en løsning må ρ ha skjæret kurven η.

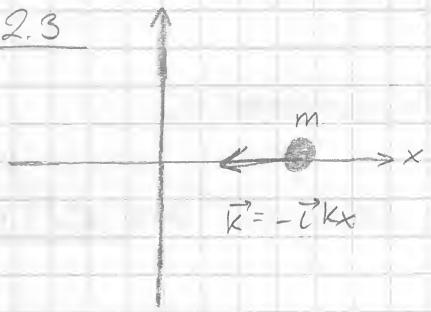


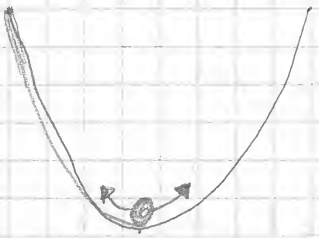
b) minste løsning: $\frac{2m}{\hbar^2}V_0a^2 \geq \pi/2$ De to kurvene tegnes inn og finner skjæringspunkt, får du kvantiserte tilstander i brønnen.

Jo flere V_0 og a^2 → større, jo flere tilstander får kvantiserte E_b når $V_0 \simeq \infty$ som vi'er kjent fra før.

c) Uendelig mange løsninger, $\rho = \infty$, slik at vi får uendelige skjæringspunkten.

2.3





a) Potensialet $V = \frac{1}{2}kx^2$

Schrödingers ligning blir det:

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + \frac{1}{2}kx^2\psi(x) = E\psi(x) \quad (I)$$

Skulle vise at $\psi_0 = e^{-\alpha x^2}$ er en løsning av denne ligningen ovenfor.

Altså:

$$\psi_0 = e^{-\alpha x^2}, \quad \psi_0' = -2x\alpha e^{-\alpha x^2}$$

$$\psi_0'' = -2\alpha e^{-\alpha x^2} - 2x\alpha\left(-2\alpha x e^{-\alpha x^2}\right)$$

$$= -2\alpha e^{-\alpha x^2}\left(1 - 2\alpha x^2\right) = e^{-\alpha x^2}\left(4\alpha^2x^2 - 2\alpha\right)$$

innsatt av ψ_0 og ψ_0'' i (I):

$$\frac{-\hbar^2}{2m}\left(4\alpha^2x^2 - 2\alpha\right)e^{-\alpha x^2} + \frac{1}{2}kx^2e^{-\alpha x^2} = E_0e^{-\alpha x^2}$$

$$\frac{-\hbar^2}{2m}\left(4\alpha^2x^2 - 2\alpha\right) + \frac{1}{2}kx^2 = E_0$$

α og E er ukjente størrelser for alle verdier av x så skal dette være oppfylt.

$$\frac{-\hbar^2}{2m}4\alpha^2x^2 + \frac{\hbar^2}{2m}2\alpha + \frac{1}{2}kx^2 = E_0$$

$\underbrace{(a-b)}_{0}x^2 + \underbrace{(c-d)}_{0} = 0$ $\qquad \underbrace{\left(\frac{1}{2}k - \frac{\hbar^2}{m}2\alpha^2\right)}_{0}x^2 + \frac{\hbar^2}{m}\alpha = E_0$

i) $-\frac{\hbar^2}{m}2\alpha^2 + \frac{1}{2}k = 0$ $\qquad$ ii) $\frac{\hbar^2}{m}\alpha = E$

Fra i) får vi: $\frac{\hbar^2}{m}2\alpha^2 = \frac{1}{2}k \Leftrightarrow \alpha^2 = \frac{mk}{\hbar^2 4} \Leftrightarrow \alpha = \frac{\sqrt{mk}}{\hbar 2}$, $\omega = \sqrt{\frac{k}{m}}$

$$\alpha = \frac{\sqrt{m(m\omega^2)}}{2\hbar} = \frac{m\omega}{2\hbar}$$

Fra ii) får vi: $\frac{\hbar^2}{m}\alpha = E_0 \Leftrightarrow E_0 = \frac{\hbar^2}{m}\frac{m\omega}{2\hbar} = \frac{\hbar\omega}{2}$ egenverdien

$\psi_0(x) = e^{-\left(\frac{m\omega}{2\hbar}\right)x^2}$ egentilstand

b) $\psi_{0(x)_{norm}} = C_0 e^{-\alpha x^2}$, skal finne normaliseringskonstanten C_0.

49, s 159

$$\int_{-\infty}^{\infty} |\psi_0|^2 dx = 1 \Rightarrow C_0^2 \int_{-\infty}^{\infty} e^{-2\alpha x^2} dx = 2C_0^2 \int_0^{\infty} e^{-2\alpha x^2} dx \Rightarrow \underline{C_0^2 \sqrt{\frac{\pi}{2\alpha}} = 1}$$

$$\Rightarrow \underline{C^2 = \sqrt{\frac{2\alpha}{\pi}} = \sqrt{\frac{2m\omega}{2\hbar\pi}} = \sqrt{\frac{m\omega}{\hbar\pi}}}$$

$$\Rightarrow \underline{C_0 = \sqrt{\sqrt{\frac{m\omega}{\hbar\pi}}} = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4}}$$

Slikat løsningen skrives på formen:

$$\underline{\psi_0(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} e^{-\left(\frac{m\omega}{2\hbar}\right)x^2}}$$

$$\langle \hat{x} \rangle = \int_{-\infty}^{\infty} \psi^* \hat{x} \psi \, dx = C_0^2 \int_{-\infty}^{\infty} e^{-\alpha x^2} x e^{-\alpha x^2} dx = C_0^2 \int_{-\infty}^{\infty} x e^{-2\alpha x^2} dx = \underline{0}$$

(antisymmetrisk · symmetrisk)

$$\langle \hat{x}^2 \rangle = \int_{-\infty}^{\infty} \psi^* \hat{x}^2 \psi \, dx = C_0^2 \int_{-\infty}^{\infty} x^2 e^{-2\alpha x^2} dx$$

$$= C_0^2 \Gamma\left(\tfrac{3}{2}\right)(2\alpha)^{-3/2} = C_0^2 \tfrac{1}{2}\sqrt{\pi}\left(\frac{\hbar}{m\omega}\right)^{3/2}$$

$$= \left(\frac{m\omega}{\hbar\pi}\right)^{1/2} \cdot \tfrac{1}{2}\sqrt{\pi}\left(\frac{\hbar}{m\omega}\right)\left(\frac{\hbar}{m\omega}\right)^{1/2} = \underline{\frac{1}{2}\left(\frac{\hbar}{m\omega}\right)}$$

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2} = \underline{\sqrt{\frac{1}{2}\left(\frac{\hbar}{m\omega}\right)}}$$

c)

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\langle \hat{p} \rangle = \int_{-\infty}^{\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi \, dx = C_0^2 \int_{-\infty}^{\infty} e^{-\alpha x^2} \left(-i\hbar \frac{\partial}{\partial x} e^{-\alpha x^2}\right) dx$$

(like · odde)

$$= C_0^2 \int_{-\infty}^{\infty} e^{-\alpha x^2} \left(-i\hbar\, e^{-\alpha x^2} (-2x\alpha)\right) dx = C_0^2 2\alpha i\hbar \int_{-\infty}^{\infty} e^{-2\alpha x^2} x \, dx = \underline{0}$$

$$\langle \hat{p}^2 \rangle = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

$$\langle \hat{p}^2 \rangle = \int_{-\infty}^{\infty} \psi^* \hat{p}^2 \psi \, dx = \int_{-\infty}^{\infty} C_0 e^{-\alpha x^2} \left(-\hbar^2 \frac{\partial^2}{\partial x^2} C_0 e^{-\alpha x^2}\right) dx$$

$$= C_0^2 \int_{-\infty}^{\infty} e^{-\alpha x^2} (-\hbar^2)(4\alpha^2 x^2 - 2\alpha) e^{-\alpha x^2} \, dx$$

$$= C_0^2 \int_{-\infty}^{\infty} e^{-2\alpha x^2} (\hbar^2)(2\alpha - 4\alpha^2 x^2) \, dx$$

$$= -4\alpha\hbar^2 C_0^2 \int_{-\infty}^{\infty} e^{-2\alpha x^2} x^2 \, dx + 2\alpha\hbar^2 C_0^2 \int_{-\infty}^{\infty} e^{-2\alpha x^2} dx$$

$$= -4\alpha\hbar^2 C_0^2 (2\alpha)^{-3/2} \frac{1}{2}\sqrt{\pi} + 2\alpha\hbar^2 C_0^2 (2\alpha)^{-1/2} \sqrt{\pi}$$

$$= \frac{1}{2}\sqrt{\pi}\,\hbar^2 C_0^2 \sqrt{2\alpha}$$

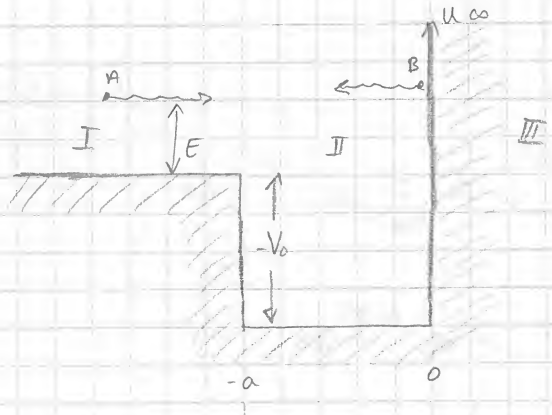
$$= \frac{1}{2}\sqrt{\pi}\,\hbar^2 \left(\frac{m\omega}{\hbar\pi}\right)^{1/2} \left(\frac{m\omega}{\hbar}\right)^{1/2}$$

$$= \underline{\frac{1}{2}\hbar m\omega}$$

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle p \rangle^2} = \underline{\sqrt{\tfrac{1}{2}\hbar m\omega}}$$

d) $$\Delta x \Delta p = \sqrt{\frac{\hbar}{2m\omega}} \sqrt{\frac{\hbar m\omega}{2}} = \sqrt{\frac{\hbar^2}{4}} = \underline{\frac{\hbar}{2}}$$

2.4



$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) + V_0(x)\psi(x) = E\,\psi(x)$$

a)

$$\frac{\partial^2\psi}{\partial x^2} = \underbrace{\frac{2m}{\hbar^2}(V-E)}_{<0}\psi$$

$$\psi_I(x) = Ae^{i\alpha x} + Be^{-i\alpha x}, \quad \alpha = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx}, \quad k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

$$\psi_{III}(x) = 0$$

Betingelsene: I $\psi_{II}(0) = \psi_{III}(0) = 0$

II $\psi_I(-a) = \psi_{II}(-a)$

III $\psi_{II}'(-a) = \psi_I'(-a)$

$\Rightarrow$: Fra betingelse I: får $\psi_{II}(0) = Ce^{ik\cdot 0} + De^{-ik\cdot 0} = C + D \Rightarrow \underline{C = -D}$

Slik at $\psi_{II}(x) = C(e^{ikx} - e^{-ikx}) = C(\cos kx + i\sin kx - \cos kx + i\sin kx)$

$= \underline{2iC\sin kx}$

$\Rightarrow$: Fra betingelse II: får $\psi_I(-a) = Ae^{i\alpha(-a)} + Be^{-i\alpha(-a)} = \underline{Ae^{-i\alpha a} + Be^{i\alpha a}}$

$\psi_{II}(-a) = 2iC\sin(k-a) = \underline{-2iC\sin ka}$

$\Rightarrow$: Fra betingelse III: får $i\alpha Ae^{i\alpha a} - i\alpha Be^{-i\alpha a} = -2iC\cos ka$

b) $\psi_I(x) = \underrightarrow{Ae^{i\alpha x}} + \underleftarrow{Be^{-i\alpha x}}$ (kompleks, går ikke mot ∞ når x settes til $\pm\infty$, for det inne holder $\sin x$ og $\cos x$, de vil bare gå i bølge form i $\pm\infty$)

$A+B=C$

$kA - kB = \alpha(A+B)$

$k-\alpha = (k+\alpha)\frac{B}{A}$

$\frac{B}{A} =$

$\rightarrow$

$\psi_{II} = 2ic \sin kx$

$\psi_I = Ae^{i\alpha x} + Be^{-i\alpha x}$

$\psi_{II} = Ce^{ikx} + De^{-ikx}$

$\psi_{III} = 0$

$C + D = 0$

$ikk - ik D = 0$

$\frac{C}{D} = \frac{k}{k} = 1$ $\qquad \underline{\underline{\left(\frac{D}{C}\right)^2 = 1}}$

$\rightarrow$

i) $II \cdot \alpha + III = 2\alpha A e^{-i\alpha a} = -2i\alpha C \sin ka + 2ick \cos ka$

ii) $II\alpha - III: \quad 2\alpha B e^{i\alpha a} = -2i\alpha c \sin ka - 2ck \cos ka$

$$\frac{i}{ii} = \frac{A}{B} = \frac{(-i\alpha \sin ka + k\cos ka)}{(-i\alpha \sin ka - k\cos ka)} \cdot e^{2i\alpha a}$$

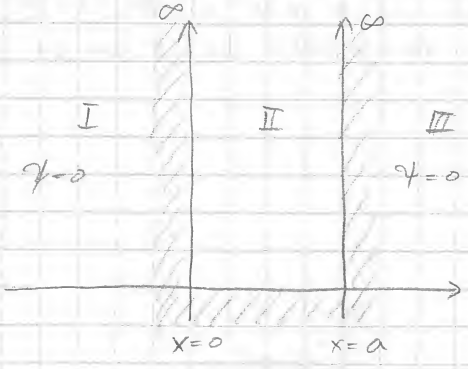
$$\left(\frac{A}{B}\right)^* \left(\frac{A}{B}\right) = \frac{(i\alpha \sin ka + k\cos ka)(-i\alpha \sin ka + k\cos ka)}{(i\alpha \sin ka - k\cos ka)(-i\alpha \sin ka - k\cos ka)} = \underline{\underline{1}}$$

$\left|\frac{A}{B}\right|^2 = 1$ $\qquad$ Refleksjon $R = \left|\frac{B}{A}\right|^2 = \underline{\underline{1}}$

c)

d) En fri partikkel har hvilket som helst energi

2.5



a) $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + 0\cdot\psi = E\cdot\psi$$

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

$$\underline{\psi'' = -k^2\psi}$$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$\left.\begin{array}{l}\psi(x=0) \quad A+B=0 \\ \underline{B=-A}\end{array}\right\} \quad \psi = A\cos kx + iA\sin kx - A\cos kx + iA\sin kx$$

$$= \underline{2iA\sin kx}$$

$$\underset{\substack{\downarrow \\ C}}{2iA}\,\underbrace{\sin(ka)}_{n\pi} = 0 \quad , n = 0,1,2,\ldots$$

$$(ka)^2 = (n\pi)^2$$

$$\frac{2mE}{\hbar^2}a^2 = n^2\pi^2 \longrightarrow E = \frac{(n\hbar/a^2)^2}{2m}n^2 = \underline{\underline{\frac{p^2}{2m}n^2}}$$

$$\int_0^a (C\psi)^2 dx = 1 \longrightarrow C = \sqrt{\frac{2}{a}} \qquad \underline{\underline{\psi = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}}$$

b) $$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi}{a}x \qquad \psi_m(x) = \sqrt{\frac{2}{a}}\sin\frac{m\pi}{a}x \qquad m \neq n$$

Hvis: $m=n$: $$\int\left(\frac{2}{a}\right)^2\sin^2\left(\frac{n\pi}{a}x\right)dx = \underline{\underline{1}}$$

Hvis: $m \neq n$:

$$\left(\sqrt{\frac{2}{a}}\right)^2\int_0^a \sin\frac{n\pi}{a}x\,\sin\frac{m\pi}{a}x\,dx$$

$$= \frac{2}{a}\int_0^a \frac{1}{2}\left\{\cos\left(\frac{n\pi - m\pi}{a}\right)x - \cos\left(\frac{n\pi + m\pi}{a}\right)x\right\}dx$$

$$= \frac{2}{a}\left[\sin\frac{(n-m)\pi}{a}x\,\frac{a}{(n-m)\pi} - \sin\frac{(n+m)\pi}{a}x\,\frac{a}{(n+m)\pi}\right]_0^a = 0$$

c) $\langle p \rangle = \int_0^a \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \psi^* dx = \int_0^a \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) - i\hbar \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) \frac{n\pi}{a} dx$

$= \frac{-2i\hbar n\pi}{a^2} \int_0^a \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{n\pi x}{a}\right) dx = \frac{-2i\hbar n\pi}{a^2} \int_0^a \frac{1}{2} \sin\left(2\frac{n\pi x}{a}\right) dx$

$= \frac{-i\hbar n\pi}{a^2} \int_0^a \sin\left(\frac{2n\pi x}{a}\right) dx = \frac{-i\hbar n\pi}{a^2} \left[-\frac{1}{2} \cos\left(\frac{2\pi x n}{a}\right)\right]_0^a$

$= \frac{-i\hbar n\pi}{a^2} \left[-\frac{1}{2} \cos\left(\frac{2\pi n a}{a}\right) + \frac{1}{2} \cos(0)\right] = \frac{-i\hbar n\pi}{a^2} \left[-\frac{1}{2}(-1) + \frac{1}{2}\right] = \underline{\frac{-i\hbar n\pi}{a^2}}$

d) $\psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$, $E = \frac{n^2\hbar^2\pi^2}{2ma^2}$

$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$ og $\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle}$

$E = \frac{p^2}{2m}$ $\langle \hat{E} \rangle = \frac{\langle \hat{p}^2 \rangle}{2m} = \frac{(\Delta p)^2}{2m}$ $\langle \hat{p}^2 \rangle = (\Delta p)^2$

$\Delta x \Delta p \geq \frac{\hbar}{2} \Leftrightarrow \Delta p \geq \frac{\hbar}{2\Delta x}$ $\rightarrow \Psi \rightarrow$ $\langle \hat{E} \rangle = \frac{\hbar^2}{2m(2\Delta x)^2} = \frac{\hbar^2}{8m\Delta x^2}$

$\Delta x = \sqrt{\left(\frac{1}{12} - \frac{1}{2\pi^2}\right)} a$ Det gir : $\langle \hat{E} \rangle = \frac{\hbar^2}{2ma^2\left(\frac{1}{3} - \frac{2}{\pi^2}\right)}$

SLIK AT :

$E \geq \langle \hat{E} \rangle \Leftrightarrow \frac{\hbar^2\pi^2}{2ma^2} \geq \frac{\hbar^2}{2ma^2(1/3 - 2/\pi^2)}$

$\pi^2 \geq \frac{1}{(1/3 - 2/\pi^2)}$

$\underline{\frac{\pi^2}{3} - 2 \geq 1}$ ok.

e) $\hat{P} e^{i(kx-\omega t)} = -i\hbar \frac{\partial}{\partial x} e^{i(kx-\omega t)} = \hbar k e^{i(kx-\omega t)}$

$= -i\hbar \frac{\partial}{\partial x} \left(\sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \right) = \hat{p} \psi$

$= -i\hbar \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi}{a} x\right) \frac{n\pi}{a}$ Vi har ødelagt sinus funksjon, altså ikke prøvd, ikke egenfunksjon.

ELLER:

$\psi(x) = A(e^{ikx} - e^{ikx})$

$= -i\hbar \frac{\partial}{\partial x} \psi(x) = A\left(ike^{ikx} + ike^{-ikx}\right) ik$

R feil, ødelagt, ikke prøvd en egenverdi

Operator x ψ = tall x ψ

f) finne operator for tiden.

Impuls operatoren: $P e^{i(kx-\omega t)} = -i\hbar \frac{\partial}{\partial x} e^{i(kx-\omega t)}$

Kanskje noe tilsvarende for t?

For plan bølge: $E = \hbar\omega$ Energi operatoren

$+i\hbar \frac{\partial}{\partial t} e^{i(kx-\omega t)} = i\hbar (i\omega) e^{i(kx-\omega t)}$

$= \hbar\omega e^{i(kx-\omega t)}$

anvendt på en plan bølge.

$+i\hbar \frac{\partial}{\partial t} e^{i(kx-\omega t)} = E e^{i(kx-\omega t)}$

$\hat{H} \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t)$

$\hat{H} X(x) T(t) = i\hbar \frac{\partial}{\partial t} X(x) T(t)$

$E X(x) T(t) = i\hbar X(x) \frac{dT}{dt} T(t)$

$\int \frac{E}{i\hbar} dt = \int \frac{dT(t)}{T(t)}$

$\frac{Et}{i\hbar} = \ln T(t)$

$T(t) = e^{-\frac{iEt}{\hbar}}$

$$\Psi(x,t) = \chi_{(x)}\, T(t)$$

$$\Psi(x,t) = \sqrt{\frac{2}{a}}\, \sin\left(\frac{n\pi x}{a}\right) e^{-\frac{iEt}{\hbar}}$$

f) $$\phi(x,t=0) = \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{n\pi x}{a}\right)$$

$$= \tfrac{1}{2}\sin\left(\pi\,\frac{n+m}{a}\,x\right) + \tfrac{1}{2}\sin\left(\pi\,\frac{n-m}{a}\,x\right)$$

$$\phi(x,t) = -\tfrac{1}{2}\sin\left(\left(\frac{n+m}{a}\right)nx\right)\cdot T(t) + \frac{1}{2}\sin\left(\frac{n-m}{a}\pi x\right) T(t)$$

$$T_+(t) = e^{-iE_+t/\hbar}, \quad E_+ = \frac{(\pi\hbar/a)^2}{2m}(m+n)^2$$

$$T_-(t) = e^{-iE_-t/\hbar}, \quad E_- = \frac{(\pi\hbar/a)^2}{2m}(n-m)^2$$

$$|\phi(x,t)|^2 = \left|\frac{1}{2}\sin\left(\left(\frac{n+m}{a}\right)\pi x\right) e^{-iE_+t/\hbar} + \tfrac{1}{2}\sin\left(\left(\frac{n-m}{a}\right)\pi x\right) e^{-iE_-t/\hbar}\right|^2$$

g) $$(\mathcal{H}_{(x)} + \mathcal{H}_{(y)})\,\chi_{(x)}\Psi_{(y)} = E\,\chi_{(x)}\Psi_{(y)}$$

$$(E_x + E_y)\,\chi\Psi = E\,\chi\Psi$$

$$\underline{E_x + E_y = E}$$

$$E = E_x + E_y + E_z$$

$$= \frac{(\hbar\pi/a)^2}{2m} n_x^2 + \frac{(\hbar\pi/2a)^2}{2m} n_y^2 + \frac{(\hbar\pi/2a)^2}{2m} n_z^2$$

$$E = \frac{(\hbar\pi/a)^2}{2m}\left\{\frac{n_x^2}{1} + \frac{n_y^2}{4} + \frac{n_z^2}{4}\right\}$$

h) $$E = \frac{(\hbar\pi/a)^2}{2m}\left\{1 + \frac{1}{4} + \frac{1}{4}\right\} \quad 1.$$

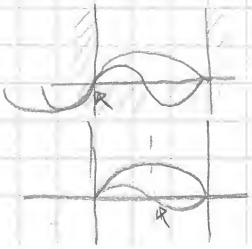
—— " —— $\left\{\frac{1}{1} + \frac{4}{4} + \frac{1}{4}\right\}$

—— " —— $\left\{\frac{1}{1} + \frac{1}{4} + \frac{4}{4}\right\}$ 2.

1) Separasjon av variablene
2) Degenarsjon

Paritet: meningsløs hvis det settes ved

Det er mening i dette her:



Skifter fortegn hvis den går gjennom origo hvis ikke, så beholdes fortegnet.

e)

Se notat (postulat 2)

$\hat{A}\psi_\nu = a_\nu \psi_\nu$ → egenfunksjon; a_ν → egenverdi, $a_\nu = 1,2,3$

Se om dette er løsningen av ovenfor:

$$\hat{p}\psi = -i\hbar \frac{\partial}{\partial x}\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) = -i\hbar\,\frac{2}{a}\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi}{a}x\right)$$

↑ ikke egenfunksjon

At den er en egentilstand for en operator, men ikke for den andre.

$$\phi_1 = e^{ikx} \Rightarrow \hat{p}\phi_1 = -i\hbar\frac{\partial}{\partial x}e^{ikx} = -i\hbar(-ik)e^{ikx} = \hbar k_1 e^{ik_1 x}$$

$$\Phi = C_1\psi_1 + C_2\psi_2 + \cdots + \sum_\nu C_\nu \psi_\nu$$

$$\psi(x) = \sqrt{\frac{2}{a}}\,\frac{1}{2i}\left(e^{\frac{in\pi x}{a}} - e^{\frac{-in\pi x}{a}}\right)$$

$\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

$e^{\frac{in\pi x}{a}} \rightarrow p = \frac{n\hbar\pi}{a}$; $e^{\frac{-in\pi x}{a}} \rightarrow p = -\frac{n\hbar\pi}{a}$

to verdier: $p_n = \pm\frac{n\hbar\pi}{a}$ $\quad |C_\nu|^2 \rightarrow 50\%$ hver av dem → ←

f)

$t = 0$

$$\Phi(x,0) = \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{a}x\right)$$

$\Phi(x,t) = ?$ $\quad \rightarrow \Phi(x,0) = \frac{1}{2}\sin\left(\frac{m+n}{a}\pi x\right) + \frac{1}{2}\sin\left(\frac{m-n}{a}\pi x\right)$

$$E_1 = \frac{(m+n)^2\hbar^2\pi^2}{2ma^2} \qquad E_2 = \frac{(m-n)^2\hbar^2\pi^2}{2ma^2}$$

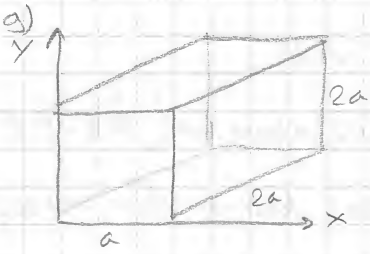
$$i\hbar\frac{\partial}{\partial t}\Phi(x,t) = \hat{H}\Phi(x,t)$$

$$\Phi(x,t) = \sum_n C_n\psi_n(x,0)\,e^{-\frac{i}{\hbar}E_n t}$$

$$\Phi(x,0) = \sum_n C_n\psi_n(x,0)$$

$$\Phi(x,t) = \frac{1}{2}\sin\left(\frac{m+n}{a}\pi x\right)e^{-i/\hbar E_1 t} + \frac{1}{2}\sin\left(\frac{m-n}{a}\pi x\right)e^{-i/\hbar E_2 t}$$

a)



$V=0 \quad 0<x<a \;,\; 0<y<2a \;,\; 0<z<2a$

$V=\infty$ ellers

$$\frac{-\hbar^2}{2m}\nabla\psi = E\psi(x,y,z) \qquad E = E_1 + E_2 + E_3$$

$$\psi = \psi_1(x)\,\psi_2(y)\,\psi_3(z)$$

$$\frac{-\hbar^2}{2m}\left(\psi_2(y)\,\psi_3(z)\frac{\partial^2\psi_1(x)}{\partial x^2} + \psi_1(y)\,\psi_3(z)\frac{\partial^2\psi_2(y)}{\partial y^2} + \psi_1(x)\,\psi_2(y)\frac{\partial^2\psi_3(z)}{\partial z^2}\right) = E\,\psi_1\psi_2\psi_3$$

$$\Rightarrow \frac{-\hbar^2}{2m}\left(\frac{1}{\psi_1}\frac{\partial^2\psi_1}{\partial x^2} + \frac{1}{\psi_2}\frac{\partial^2\psi_2}{\partial y^2} + \frac{1}{\psi_3}\frac{\partial^2\psi_3}{\partial z^2}\right) = E$$

$$\frac{-\hbar^2}{2m}\frac{1}{\psi_1(x)}\frac{\partial^2\psi_1(x)}{\partial x^2} = E_1 \;,\quad \frac{-\hbar^2}{2m}\frac{1}{\psi_2(y)}\frac{\partial^2\psi_2(y)}{\partial y^2} = E_2 \;,\quad \frac{-\hbar^2}{2m}\frac{1}{\psi_3(z)}\frac{\partial^2\psi_3(z)}{\partial z^2} = E_3$$

$$\psi = A\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi y}{2a}\right)\sin\left(\frac{n_3\pi z}{2a}\right)$$

$$A^2\int_0^{2a}\int_0^{2a}\int_0^{a}\sin^2\left(\frac{n_1\pi x}{a}\right)\sin^2\left(\frac{n_2\pi y}{2a}\right)\sin^2\left(\frac{n_3\pi z}{2a}\right)dx\,dy\,dz$$

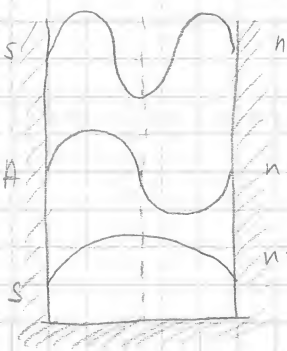
$$\Rightarrow \underline{\underline{A = \sqrt{\frac{2}{a^3}}}}$$

$$E = \frac{n_1^2\hbar^2\pi^2}{2ma^2} + \frac{n_2^2\hbar^2\pi^2}{2m(2a)^2} + \frac{n_3^2\hbar^2\pi^2}{2m(2a)^2} = \underline{\underline{\frac{\hbar^2\pi^2}{2ma^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{4}\right)}}$$

b)

Energi	n_1	n_2	n_3	degenarasjon	Pariket
$E_1 = 3/2\,E_0$	1	1	1	1	S (+1)
$E_2 = 9/4\,E_0$	(1,2,1) ,	(1,1,2)		2	A (-1)
$E_3 = 3E_0$	1	2	2	1	S (+1)

Regler for pariket:



$\Psi = \psi_1(x)\,\psi_2(y)\,\psi_3(z)$

$A \cdot A = S$

$S \cdot S = S$

$A \cdot S = A$

2.6

En kvantemekanisk kommutator er definert ved

$$[\hat{A},\hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} \qquad \text{hvis og bare hvis} \left[\hat{A}\hat{B} - \hat{B}\hat{A} = 0\right]$$

Dersom to fysiske størrelser $\hat{A}$ og $\hat{B}$ samtidig skal kunne ha veldefinerte verdier så må systemet være i en felles egentilstand for de to tilhørende operatorer $\hat{A}$ og $\hat{B}$

$$\hat{A}\psi_n = a\psi_n \quad ; \quad \hat{B}\psi_n = b\psi_n \qquad (8)$$

Ved å operere på de første av disse likningene med $\hat{A}$ fås

$$\hat{A}\hat{B}\psi_n = b\hat{A}\psi_n = ba\psi_n$$

tilsvarende gir operatoren med $\hat{B}$ også den siste likningen (8)

$$\hat{B}\hat{A}\psi_n = a\hat{B}\psi_n = ba\psi_n$$

Subtraksjon av de to siste likningen gir

$$\underline{(\hat{A}\hat{B} - \hat{B}\hat{A})\psi_n = 0}$$

skulle vise:

$$[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B}$$

(1) $[\hat{A}\hat{B},\hat{C}] = \underline{\hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B}}$ Q.E.D

(2) $\hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B} = \hat{A}(\hat{B}\hat{C} - \hat{C}\hat{B}) + (\hat{A}\hat{C} - \hat{C}\hat{B})\hat{B}$

$= \hat{A}\hat{B}\hat{C} - \hat{A}\hat{C}\hat{B} + \hat{A}\hat{C}\hat{B} - \hat{C}\hat{A}\hat{B}$

$= \underline{\hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B}}$ Q.E.D

Den kvantemekaniske operatoren for en partikkels banespinn er definert ved

$$\hat{\vec{L}} = \vec{r} \times \hat{\vec{p}}$$

Vise at vi får dette:

$$\hat{L}_x = y\hat{P}_z - z\hat{P}_y \quad , \quad \hat{L}_y = z\hat{P}_x - x\hat{P}_z \quad , \quad \hat{L}_z = x\hat{P}_y - y\hat{P}_x$$

Jo!

$$\hat{\vec{L}} = \vec{r} \times \hat{\vec{p}} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ \hat{P}_x & \hat{P}_y & \hat{P}_z \end{vmatrix} = \vec{i}(y\hat{P}_z - z\hat{P}_y) - \vec{j}(x\hat{P}_z - z\hat{P}_x) + \vec{k}(x\hat{P}_y - y\hat{P}_x)$$

$$\hat{L}_x \qquad\qquad \hat{L}_y \qquad\qquad \hat{L}_z$$

Skulle utlede følgende kommutator relasjoner

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z \quad , [\hat{L}_y, \hat{L}_z] = i\hbar \hat{L}_x \quad , [\hat{L}_z, \hat{L}_x] = i\hbar \hat{L}_y$$

Jo!

$$[\hat{L}_x, \hat{L}_y] = [y\hat{p}_z - z\hat{p}_y][z\hat{p}_x - x\hat{p}_z]$$

$$= (y\hat{p}_z - z\hat{p}_y)(z\hat{p}_x - x\hat{p}_z) - (z\hat{p}_x - x\hat{p}_z)(y\hat{p}_z - z\hat{p}_y)$$

$$= y\hat{p}_z z\hat{p}_x - y\hat{p}_z x\hat{p}_z - z\hat{p}_y z\hat{p}_x + z\hat{p}_y x\hat{p}_z - z\hat{p}_x y\hat{p}_z + z\hat{p}_x z\hat{p}_y + x\hat{p}_z y\hat{p}_z - x\hat{p}_z z\hat{p}_y$$

$$= y\hat{p}_z z\hat{p}_x - x\hat{p}_z z\hat{p}_y$$

$$= y\left(-i\hbar \frac{\partial}{\partial z}(z)\right)\hat{p}_x - x\left(-i\hbar \frac{\partial}{\partial z}(z)\hat{p}_y\right)$$

$$= y(-i\hbar)\hat{p}_x + x(i\hbar)\hat{p}_y$$

$$= i\hbar(x\hat{p}_y - y\hat{p}_x) = \underline{\underline{i\hbar \hat{L}_z}}$$

På grunn av symmetri kan de andre utledes ganske greit.

2.8

Vi skal igjen studere systemet som er beskrevet i 2.5 og antar ved tiden $t=0$ er det i tilstanden

$$\Psi(x,0) = \sqrt{\frac{1}{2}}(\psi_1(x) + \psi_2(x))$$

a) Vise at den er normert til 1, bestemme $\Psi(x,t)$, forventningsverdi $\langle x \rangle$ som funksjon av t.

Jo! Partikkel i boks problemet igjen.

Normering:

$$\int_0^a \psi^*(x)\psi(x)dx = \frac{1}{2}\int(\psi_1(x) + \psi_2(x))^2 dx$$

$$= \frac{1}{2}\left(\int \psi_1(x)^2 dx + \int(\psi_2(x))^2 dx + \int \psi_1(x)\psi_2(x)\,dx\right)$$

$$= \frac{1}{2}(1 + 1 + 0) = \underline{1}$$

$\Psi(x,t)$ fra T.A.S.L

$$\underline{\Psi(x,t) = \sqrt{\frac{1}{2}}\left(\psi_1(x)e^{-\frac{iE_1 t}{\hbar}} + \psi_2(x)e^{-\frac{iE_2 t}{\hbar}}\right)dx}$$

2.6

En kvante mekanisk kommutator er definert ved

$$[\hat{A},\hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} \qquad \text{hvis og bare hvis} \quad [\hat{A}\hat{B} - \hat{B}\hat{A} = 0]$$

Dersom to fysiske størrelser $\hat{A}$ og $\hat{B}$ samtidig skal kunne ha veldefinerte verdier så må systemet være i en felles egentilstand for de to tilhørende operatorer $\hat{A}$ og $\hat{B}$

$$\hat{A}\psi_n = a\psi_n \quad ; \quad \hat{B}\psi_n = b\psi_n \qquad (8)$$

Ved å operere på de første av disse likningene med $\hat{A}$ fås

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tilsvarende gir operatoren med $\hat{B}$ også den siste likningen (8)

$$\hat{B}\hat{A}\psi_n = a\hat{B}\psi_n = ba\psi_n$$

Subtraksjon av de to siste likningen gir

$$\underline{(\hat{A}\hat{B} - \hat{B}\hat{A})\psi_n = 0}$$

skulle vise:

$$[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] - [\hat{A},\hat{C}]\hat{B}$$

(1) $[\hat{A}\hat{B},\hat{C}] = \underline{\hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B}}$

(2) $\hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B} = \hat{A}(\hat{B}\hat{C} - \hat{C}\hat{B}) + (\hat{A}\hat{C} - \hat{C}\hat{A})\hat{B}$

$$\vec{L} = \vec{r}\times\vec{p} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ \hat{p}_x & \hat{p}_y & \hat{p}_z \end{vmatrix} = \vec{i}(y\hat{p}_z - z\hat{p}_y) - \vec{j}(x\hat{p}_z - z\hat{p}_x) + \vec{k}(x\hat{p}_y - y\hat{p}_x)$$

$$\hat{L}_x \qquad \hat{L}_y \qquad \hat{L}_z$$

Forventningsverdien $\langle \hat{x} \rangle$

$$\langle \hat{x} \rangle = \int_0^a \Psi^*(x,t) \cdot \Psi(x,t)\, dx$$

$$= \frac{1}{2} \int_0^a \left(\psi_1(x) e^{\frac{iE_1 t}{\hbar}} + \psi_2(x) e^{\frac{iE_2 t}{\hbar}} \right) \cdot \left(\psi_1(x) e^{-\frac{iE_1 t}{\hbar}} + \psi_2(x) e^{-\frac{iE_2 t}{\hbar}} \right) dx$$

$$= \frac{1}{2} \int_0^a \left(\psi_1(x)\, \psi_1(x) + \psi_2(x) \cdot \psi_2(x) \right) dx + \frac{1}{2} \int_0^a \left(\psi_1(x) \cdot \psi_2(x) e^{i\frac{(E_1-E_2)t}{\hbar}} + \psi_1(x)\, \psi_2(x) e^{-i\frac{(E_1-E_2)t}{\hbar}} \right) dx$$

$$= I_1 + I_2$$

Finner først I_1 ; $\left(\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right) \right)$

b) $\langle p \rangle = \frac{2}{L} \frac{1}{2} \int_{x=0}^{L} \left(\sin\frac{\pi x}{L} e^{\frac{i t}{\hbar}(E_1)} + \sin\frac{2\pi x}{L} e^{\frac{i t E_2}{\hbar}} \right) \frac{\hbar}{i} \frac{\partial}{\partial x} \left(\sin\frac{\pi x}{L} e^{-\frac{i t E_1}{\hbar}} + \sin\left(\frac{2\pi x}{L}\right) e^{-\frac{i t E_2}{\hbar}} \right) dx$

$$= \frac{1}{L} \frac{\hbar}{i} \int_0^L \sin\frac{\pi x}{L} \frac{d}{dx}\left(\sin\frac{\pi x}{L}\right) e^{\frac{i t}{\hbar}(E_1 - E_1)} dx \longrightarrow 0$$

$$+ \frac{1}{L} \frac{\hbar}{i} \int_0^L \sin\frac{\pi x}{L} \cos\left(\frac{2\pi x}{L}\right) \frac{2\pi}{L} e^{\frac{i t}{\hbar}(E_1 - E_2)} dx$$

$$+ \frac{1}{L} \frac{\hbar}{i} \int_0^L \sin\left(\frac{2\pi x}{L}\right) \frac{d}{dx}\left(\sin\frac{2\pi x}{L}\right) e^{\frac{i t}{\hbar}(E_2 - E_2)} dx \longrightarrow 0$$

$$+ \frac{1}{L} \frac{\hbar}{i} \int_0^L \sin\frac{2\pi x}{L} \cos\left(\frac{\pi x}{L}\right) \frac{\pi}{L} e^{\frac{i t}{\hbar}(E_2 - E_1)} dx$$

$$= \frac{\hbar}{Li} \frac{2\pi}{L} e^{\frac{i\hbar}{t}(E_1 - E_1)} \int_0^L \sin\frac{\pi x}{L} \cos\frac{2\pi x}{L} dx$$

$$+ \frac{\hbar}{Li} \frac{\pi}{L} e^{\frac{i t}{\hbar}(E_2 - E_1)} \int_0^L \sin\frac{2\pi x}{L} \cos\frac{\pi x}{L} dx$$

$$= \frac{2\pi\hbar}{L^2 i} e^{\frac{i t}{\hbar}(E_1 - E_1)} \left(-\frac{2L}{3\pi}\right) + \frac{\pi\hbar}{iL^2} e^{\frac{i t}{\hbar}(E_2 - E_1)} \left(\frac{4}{3}\frac{L}{\pi}\right)$$

$$= -\frac{2\pi\hbar}{iL^2} \frac{2L}{3\pi} \left(\cos\frac{t}{\hbar}(E_1 - E_2) + i \sin\frac{t}{\hbar}(E_1 - E_2) \right) + \frac{4\pi\hbar L}{iL^2 3\pi} \left(\cos\frac{t}{\hbar}(E_2 - E_1) + i \sin\frac{t}{\hbar}(E_2 - E_1) \right)$$

$$= \frac{8}{3} \frac{\hbar}{L} \sin\frac{t}{\hbar}(E_2 - E_1)$$

b) fort:

$$m\frac{d}{dt}\langle x\rangle = m\frac{d}{dt}\int_0^L \frac{2}{L}\left(\frac{1}{\sqrt{2}}\right)^2\left\{\sin\frac{\pi x}{L}e^{\frac{i\hbar E_1}{\hbar}} + \sin\frac{2\pi x}{L}e^{\frac{itE_2}{\hbar}}\right\}\cdot x\cdot$$

$$\left\{\sin\frac{\pi x}{L}e^{\frac{-itE_1}{\hbar}} + \sin\frac{2\pi x}{L}e^{\frac{-itE_2}{\hbar}}\right\}dx$$

$$= \frac{d}{dt}\underbrace{\frac{m}{L}\int_0^L\left(\sin^2\left(\frac{\pi x}{L}\right)x + \sin^2\left(\frac{2\pi x}{L}\right)x\right)dx}_{\frac{d}{dt}(\text{tall})=0} + \frac{m}{L}\frac{d}{dt}\int_0^L x\sin\frac{\pi x}{L}\sin\frac{2\pi x}{L}\left[e^{\frac{it}{\hbar}(E_1-E_2)} + e^{\frac{it}{\hbar}(E_2-E_1)}\right]dx$$

$$= \frac{im}{\hbar L}\left(\frac{i}{\hbar}(E_1-E_2)e^{\frac{it}{\hbar}(E_1-E_2)} + \frac{i}{\hbar}(E_2-E_1)e^{\frac{it}{\hbar}(E_2-E_1)}\right)\cdot\int_0^L x\cdot\frac{1}{2}\left(\cos\left(\frac{2\pi}{L}-\frac{\pi}{L}\right)x - \cos\left(\frac{2\pi}{L}+\frac{\pi}{L}\right)x\right)dx$$

$$= \frac{mi}{\hbar L}\left\{E_1\cos\frac{t}{\hbar}(E_1-E_2) + E_1 i\sin\frac{t}{\hbar}(E_1-E_2) - E_2\cos\frac{t}{\hbar}[E_1-E_2) - E_2\cdot i\cdot\sin\frac{t}{\hbar}(E_1-E_2)\right.$$

$$\left. + E_2\cos\frac{t}{\hbar}(E_2-E_1) + E_2\cdot i\cdot\sin\frac{t}{\hbar}(E_2-E_1) - E_1\cos\frac{t}{\hbar}(E_2-E_1) - E_1 i\sin\frac{t}{\hbar}(E_2-E_1)\right\}\cdot$$

$$\left\{\int_0^L\left(x\frac{1}{2}\sin\left(\frac{2\pi}{L}-\frac{\pi}{L}\right)\left(\frac{2\pi}{L}-\frac{\pi}{L}\right)^{-1} \div \frac{x}{2}\sin\left(\frac{2\pi}{L}+\frac{\pi}{L}\right)\left(\frac{2\pi}{L}+\frac{\pi}{L}\right)^{-1}\right)dx \to 0\right.$$

$$\left. - \int_0^L\frac{1}{2}\frac{L}{\pi}\sin\frac{\pi x}{L}dx + \int_0^L\frac{1}{2}\frac{L}{3\pi}\sin\frac{3\pi x}{L}dx\right\}$$

$$+\frac{L}{2\pi}\left[\cos\frac{\pi x}{L}\cdot\frac{L}{\pi} - \frac{L}{6\pi}\right]_0^L \qquad -\frac{L}{6\pi}\left[\cos\frac{3\pi x}{L}\cdot\frac{L}{3\pi}\right]_0^L$$

Totalt: $\underline{\underline{\frac{8\hbar}{3L}\sin\left(\frac{t}{\hbar}(E_2-E_1)\right)}}$

c) Hvor generelt er dette !? TRIKS:

$$\frac{d}{dt}\langle\hat{Q}\rangle = \frac{-i}{\hbar}\langle[\hat{Q},\hat{H}]\rangle \qquad [\hat{Q},\hat{H}] \equiv \hat{Q}\hat{H} - \hat{H}\hat{Q}$$

$$\frac{d}{dt}\langle Q\rangle = \frac{d}{dt}\int\psi^*\hat{Q}\psi\,dx = \int\left(\frac{d}{dt}\psi^*\right)\hat{Q}\psi\,dx + \int\psi^*\underbrace{\frac{d\hat{Q}}{dt}}_{0,\text{ for } x \text{ og } t\ \frac{dx}{dt}=0}\psi\,dx + \int\psi^*\hat{Q}\frac{d\psi}{dt}dx$$

$$= \int \frac{1}{i\hbar} \mathcal{H}\psi^* \hat{Q}\psi dx + \int \psi^* \hat{Q} \frac{1}{i\hbar} \psi dx$$

$$= \frac{i^{-1}}{\hbar} \int \psi^* (\mathcal{H}Q - Q\mathcal{H})\psi dx = \frac{i}{\hbar} \int \psi^* [\hat{H}, \hat{Q}] \psi dx \quad (1)$$

$$m \frac{d}{dt}\langle x \rangle = \langle p \rangle \qquad \frac{d}{dt}\langle x \rangle = \frac{i}{\hbar} \int \psi^* \{\hat{\mathcal{H}}\hat{x} - \hat{x}\hat{\mathcal{H}}\} \psi dx$$

$$\boxed{m \frac{d}{dt} x = p \Leftrightarrow mv = p}$$

$$\hat{\mathcal{H}} = \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + \hat{V}(x)$$

$\hat{x}V(x) - V(x)\hat{x} = 0$
den kommer med x.

$$(1) = \frac{i}{\hbar} \left\{ \psi^* \left\{ \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} x - x \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} \right\} \psi dx \right\}$$

$$= i/\hbar \left(\frac{-\hbar^2}{2m}\right) \int \psi^* \frac{2d}{dx} \psi dx = \underline{\frac{1}{m}\langle p \rangle}$$

$$\frac{d}{dx}\left(\frac{d}{dx} x\psi\right) = \frac{d}{dx}\left(\psi + x\psi'\right)$$
$$= \psi' + \psi' + x\psi'' - x\psi''$$
$$= \underline{2\psi'}$$

$$\boxed{\begin{array}{c} m \frac{d}{dt}\langle x \rangle = \langle p \rangle \\ \# \\ m \frac{d}{dt}\langle \hat{x} \rangle = \langle \hat{p} \rangle \end{array}}$$

KRAFT $V(x) = \frac{1}{2} K x^2$ } er dette generelt

$\vec{K} = -Kx\vec{i}$

$K = ma = -\frac{d}{dt} V(x)$

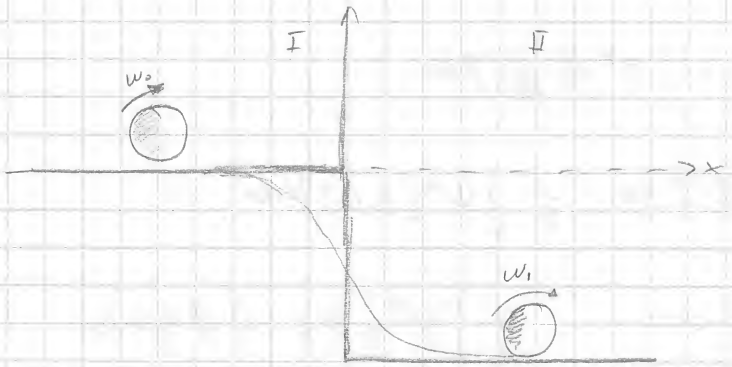
$$\text{"}ma\text{"} = \frac{d}{dt}\langle \hat{p} \rangle = \frac{i}{\hbar} \int \psi^* [\hat{H}, \hat{p}] \psi dx \quad ; \quad \hat{\mathcal{H}} = \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)$$

$$= \frac{i}{\hbar} \int \psi^* \left\{ \frac{\hbar^2}{2m} \frac{d^2}{dx^2} (-i\hbar) \frac{\partial}{\partial x} - (-i\hbar) \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - V(-i\hbar) \frac{\partial}{\partial x} + (-i\hbar) \frac{\partial}{\partial x} V \right\} \psi dx$$

$$= \int \psi^* \left(-\frac{\partial V(x)}{\partial x}\right) \psi dx = \left(-\frac{d E_{pot}}{dx}\right) = -\text{"}\frac{dV}{dt}\text{"}$$

under forutsetning at $\frac{dQ}{dt} = 0$

2.10



a) partikkelen kommer ned og får større kinetisk energi.

b) $I: \quad -\frac{\hbar^2}{2m}\frac{d^2\psi_I}{dx^2} = E\psi_I \quad \Rightarrow \quad \psi_I = Ae^{ik_1x} + Be^{-ik_1x}$

$II: \quad -\frac{\hbar^2}{2m}\frac{d^2\psi_{II}}{dx^2} = (E+V_0)\psi_{II} \quad \Rightarrow \quad \psi_{II} = Ce^{ik_2x} + De^{-ik_2x}$

$k_1^2 = \frac{2mE}{\hbar^2} \qquad k_2^2 = \frac{2m(E+V_0)}{\hbar^2}$

$D=0$ for den går ikke tilbake, fortsetter bare

c)

$\psi_I(x=0) = \psi_{II}(x=0) \quad \Rightarrow \quad A+B=C \qquad (1)$

$\psi_I'(x=0) = \psi_{II}'(x=0) \quad \Rightarrow \quad k_1(A-B) = k_2C \qquad (2)$

(1): $\Rightarrow \quad 1+\frac{B}{A} = \frac{C}{A}$ (2): $k_1\left(1-\frac{B}{A}\right) = k_2\frac{C}{A} \Leftrightarrow k_1\left(1-\frac{B}{A}\right) = k_2\left(1+\frac{B}{A}\right)$

RES: $\Rightarrow \quad \underline{\frac{B}{A} = \frac{k_1-k_2}{k_1+k_2}} \qquad \underline{\frac{C}{A} = \frac{2k_1}{k_1+k_2}}$

$\underline{\left(\frac{B}{A}\right)^2 = \left(\frac{k_2-k_1}{k_2+k_1}\right)^2} \qquad \underline{\left(\frac{C}{A}\right)^2 = \left(\frac{2k_1}{k_1+k_2}\right)^2}$

d)

$R = \frac{\text{fluks ut}}{\text{fluks inn}} = \left|\frac{B}{A}\right|^2\frac{v_1}{v_1} = \underline{\left|\frac{B}{A}\right|^2} \qquad T = \left|\frac{CV_2}{AV_1}\right|^2 = \underline{\left|\frac{C}{A}\right|^2\frac{k_2}{k_1}}$

e) $1 = R+T = \left|\frac{B}{A}\right|^2 + \left|\frac{C}{A}\right|^2\frac{k_2}{k_1} = \frac{(k_2-k_1)(k_2-k_1)}{(k_2+k_1)(k_2+k_1)} + \frac{(2k_1)^2}{(k_1+k_2)(k_1+k_2)}\frac{k_2}{k_1}$

$= \frac{(k_2-k_1)(k_2-k_1)k_1 + 4k_1^2k_2}{(k_2+k_1)(k_1+k_2)k_1}$

2.9

a)

b) $$-\frac{\hbar^2}{2m}\nabla^2\Psi(r,t) + V(\vec{r})\,\Psi(r,t) = i\hbar\frac{\partial\Psi}{\partial t}(r,t)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

c) $$\Psi(r,t) = \psi(r)\,e^{-\frac{iEt}{\hbar}}$$

h.s: $$i\hbar\frac{\partial}{\partial t}\Psi(r,t) = i\hbar\left(-\frac{iE}{\hbar}\right)\Psi$$

v.s: $$e^{-\frac{iEt}{\hbar}}\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right)\psi(\vec{r})$$

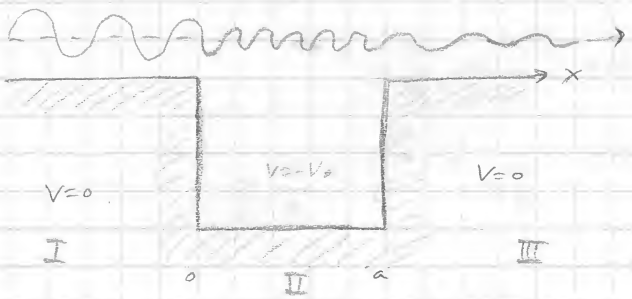
$$-i^2\hbar\frac{E}{\hbar}\psi e^{-\frac{iEt}{\hbar}} = e^{-\frac{iEt}{\hbar}}\hat{H}\psi(\vec{r})$$

$$\hat{H}\psi(\vec{r}) = E\psi(\vec{r})$$

d) $$\hat{A} = \hat{A}(\vec{r},\vec{p}) = \hat{A}(\vec{r},-i\hbar\vec{\nabla})$$

$\hat{A}\psi_\nu = a_\nu\psi_\nu$, $\nu = 1,2,3\ldots$ hvor a_ν er egenverdiene mulige målte verdier.

e)



I: $E_k = E$ $\quad\lambda = \frac{h}{p} = \frac{h}{(2mE)^{1/2}}$

II: $E_k = E + V_0$ $\quad\lambda = \frac{h}{p} = \frac{h}{(2mE+V_0)}$

III: $E_k = E$ $\quad\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

Amplitudene blir forandret, med at noen blir reflektert tilbake

1) $\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi$

$$\frac{\partial^2\psi}{\partial x^2} = \overbrace{(V-E)\frac{2m}{\hbar^2}}^{<0}\psi$$

$\Psi_I(x) = Ae^{ikx} + Be^{-ikx}$ $\qquad \Psi_{II}(x) = Ce^{ik'x} + De^{-ik'x}$

$\Psi_{III}(x) = Fe^{ikx} + Ge^{-ikx}$

$k = \sqrt{\frac{2mE}{\hbar^2}}$

$k' = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

Randkravene:

$x=0$:

i) $\Psi_I(0) = \Psi_{II}(0)$ $\qquad A+B = C+D$

ii) $\Psi_I'(0) = \Psi_{II}'(0)$ $\qquad ikA - ikB = ik'C - ik'D$

$x=a$:

iii) $\Psi_{II}(a) = \Psi_{III}(a)$ $\qquad Ce^{ik'a} + De^{-ik'a} = Fe^{ika}$

iv) $\Psi_{II}'(a) = \Psi_{III}'(a)$ $\qquad ik'Ce^{ik'a} - ik'De^{-ik'a} = ikFe^{ika}$

g)

i) $\times \frac{1}{A}$ $\qquad 1 + \frac{B}{A} = \frac{C}{A} + \frac{D}{A}$

ii) $\times \frac{1}{ikA}$ $\qquad 1 - \frac{B}{A} = \frac{k'}{k}\left(\frac{C}{A} - \frac{D}{A}\right)$

iii) $\times \frac{1}{A}$ $\qquad \frac{C}{A}e^{ik'a} + \frac{D}{A}e^{-ik'a} = \frac{F}{A}e^{-ika}$

iv) $\times \frac{1}{ika}$ $\qquad \frac{k'}{k}\left(\frac{C}{A}e^{ik'a} - \frac{D}{A}e^{-ik'a}\right) = \frac{F}{A}e^{ika}$

og $\frac{k'}{k} = \sqrt{\frac{\frac{2m(E+V_0)}{\hbar^2}}{\frac{2mE}{\hbar^2}}} = \sqrt{\frac{E+V_0}{E}} = \sqrt{1+\frac{V_0}{E}} = \lambda$

$|A|^2$, $|B|^2$, intensiteten til partiklene.

$p = \hbar k \quad \Leftrightarrow \quad k = \frac{p}{\hbar}$, bølgetallet

h) Antar $V_0 = 3E$

$$R = \frac{|B|^2}{|A|^2} = \frac{\text{antall reflektert partikler}}{\text{Antall inkommende partikler}} = \left|\frac{B}{A}\right|^2 = |x|^2$$

(i) $$1 + x = \frac{C}{A} + \frac{D}{A}$$

(ii) $$1 - x = 2\left(\frac{C}{A} - \frac{D}{A}\right)$$

(iii) $$\frac{C}{A} e^{ika'} + \frac{D}{A} e^{-ik'a} = \frac{F}{A} e^{ika}$$

(iv) $$2\left(\frac{C}{A} e^{ik'a} - \frac{D}{A} e^{-ik'a}\right) = \frac{F}{A} e^{ika}$$

$\Longrightarrow$ (iii) og (iv)

$$\frac{C}{A} e^{ika'} + \frac{D}{A} e^{-ik'a} = 2\frac{C}{A} e^{ik'a} - 2\frac{D}{A} e^{-ik'a}$$

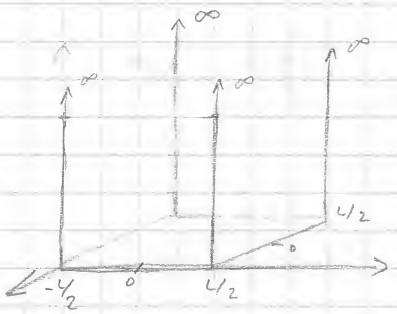
$$\frac{C}{A} = 3\frac{D}{A} e^{-2ik'a}$$

$$x = \frac{3(e^{-2ik'a} - 1)}{(-9e^{-2ik'a} + 1)}$$

$$R = |x|^2 = x^* x = \frac{9(\sin k'a)^2}{25\sin^2 k'a + 16\cos^2 k'a} = \frac{1}{\frac{25}{9} + \frac{16}{9}\cot g^2(k'a)}$$

hvor $k'a = \frac{a}{\hbar}\sqrt{8mE}$

3.1



$V(x,y) = 0$ for $|x| < L/2$ og $|y| < L/2$

$V(x,y) = \infty$ ellers

a) Hamilton operatoren for systemet,

$$\hat{\mathcal{H}} = \left\{ -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + V(x) \right\} \psi \quad , \quad V(x) = 0$$

$$\hat{\mathcal{H}} = \left\{ -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \psi \right\}$$

Vi antar det som skjer i x-retningen er upåvirket av y-retningen

$$\psi(x,y) = X(x)\,Y(y)$$

$$\left(\hat{\mathcal{H}}(x) + \hat{\mathcal{H}}(y)\right) X(x)\,Y(y) = E\,X(x)\,Y(y)$$

$$\hat{\mathcal{H}}_{(x)}\left(X(x)\,Y(y)\right) + \hat{\mathcal{H}}_y\left(X(x)\,Y(y)\right) = E\,X(x)\,Y(y) \qquad | : X(x)\,Y(y)$$

$$Y(y)\,\hat{\mathcal{H}}(x)\,X(x) + X(x)\,\hat{\mathcal{H}}(y)\,Y(y) = E\,X(x)\,Y(y)$$

$$\frac{1}{X(x)}\hat{\mathcal{H}}(x)\,X(x) + \frac{1}{Y(y)}\hat{\mathcal{H}}_y\,Y(y) = E$$

$$= E_x + E_y = E$$

$$E_x = \frac{(\pi\hbar/a)^2}{2m}\,a_x^2$$

2.16

a) $\psi_v(x,t) = e^{\left(\frac{i}{\hbar} m v \left(x - \frac{v}{2} t\right)\right)}$

$$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi_v(x,t) = i\hbar \frac{\partial}{\partial t} \psi_v(x,t)$$

V.S.:

$$\frac{-\hbar^2}{2m} \left(\frac{i}{\hbar} m v\right)^2 \psi_v(x,t) = \frac{1}{2} m v^2 \psi_v(x,t)$$

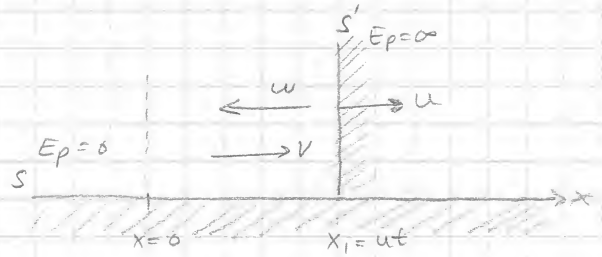
H.S.:

$$i\hbar \left(\frac{i}{\hbar} m v \left(-\frac{v}{2}\right)\right) \psi_v(x,t) = \tfrac{1}{2} m v^2 \psi_v(x,t)$$

Vise at den har fast v,

$$\hat{p}\psi_v = -i\hbar \frac{\partial}{\partial x} \psi_v = -i\hbar \frac{i}{\hbar} m v \psi_v = \underline{\underline{m v \psi_v}}$$

b)



$$E_p = \begin{cases} 0 & \text{for } x < x_1 \\ +\infty & \text{for } x > x_1 \end{cases}$$

$V > 2u$, $w = -v + 2u$

S': $u' = 0$, $v' = v - u$, $w' = -v + u$

S: $w = w' + u = \underline{\underline{-v + 2u}}$

c) $\Psi(x,t) = A\Psi_v(x,t) + B\Psi_w(x,t) \qquad x < x_1$

$\Psi_v = e^{i/\hbar\, mv(x - \frac{v}{2}t)}$

$\Psi_w = e^{i/\hbar\, mw(x - \frac{w}{2}t)}$

$x_1 = ut \qquad \Psi(x,t) = 0$

$\Psi = Ae^{i/\hbar\, mv(ut - \frac{v}{2}t)} + Be^{\frac{i}{\hbar} mw(ut - \frac{w}{2}t)} = 0$

$Ae^{i/\hbar\, mvt(u - v/2)} + Be^{i/\hbar\, mvt(u - v/2)} = 0$

$(A+B)e^{i/\hbar\, mvt(u - v/2)} = 0$

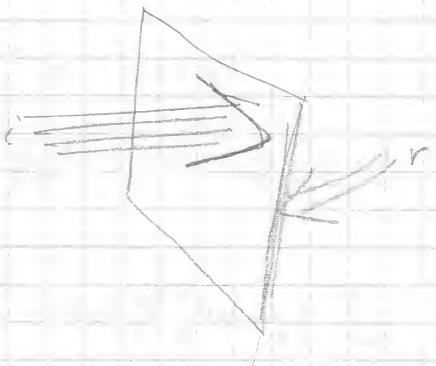
$\underline{\underline{A = -B}}$

d) antall partikler per tid (hvermott).

$\Phi_i = v|A|^2$

$\Phi_r = (v-2u)|B|^2 = (v-2u)|A|^2$

$R = \frac{\Phi_r}{\Phi_i} = \frac{(v-2u)|A|^2}{v|A|^2} = \underline{\underline{1 - \frac{2u}{v}}} < 1$



3-2

a) klassisk uttrykk for banespinnet er gitt ved

$$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} i & j & k \\ x & y & z=0 \\ P_x & P_y & P_z=0 \end{vmatrix} = i(y\cdot 0 - 0\cdot P_y) - j(x\cdot 0 - 0 P_x) + k(xP_y - yP_x)$$

$$\Rightarrow \underline{k(xP_y - yP_x) = L_z}$$

hvor $z=0$ og $P_z=0$

b) Partikkelens kinetiske energi uttrykt ved L_z.

$$L = rp\sin\theta = 1 \Leftrightarrow L = rp \Leftrightarrow p = \frac{L_z}{r}$$

$$E_k = \frac{p^2}{2m} = \underline{\frac{L_z^2}{2mr^2}}$$

c) $\hat{\mathcal{H}}\psi = E\psi$

$$\hat{\mathcal{H}} = \frac{L_z^2}{2mr^2} = \left(-i\hbar\frac{\partial}{\partial\varphi}\right)^2 \frac{1}{2mr^2} = \underline{-\frac{\hbar}{2mr^2}\frac{\partial^2}{\partial\varphi^2}} \quad \text{hvor } \hat{L}_z = \left(-i\hbar\frac{\partial}{\partial\varphi}\right)$$

d) De kravene som stilles til partikkelens bølgefunksjon er disse:

I) Kontinuerlig
II) Normerbar
III) Entydig
IV) $\frac{d\psi}{d\varphi}$ kontinuerlig

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{d\varphi^2} = E\psi \Leftrightarrow \frac{d^2\psi}{d\varphi^2} = -\frac{2mr^2E}{\hbar^2}\psi$$

$$\underline{\psi(\varphi) = Ae^{ik\varphi} + Be^{-ik\varphi}}, \quad k^2 = \frac{2mr^2E}{\hbar^2}$$

Betingelsene:

$$\psi(0) = \psi(2\pi)$$

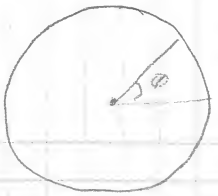
$$Ae^{ik0} + Be^{-ik0} = Ae^{ik2\pi} + Be^{-ik2\pi}$$

$$A + B = A[\cos(2k\pi) + i\sin(2k\pi)] + B[\cos(2k\pi) - i\sin(2k\pi)]$$

$$\cos(2\pi k) + i\sin(2\pi k) = \cos(2\pi k) - i\sin(2\pi k)$$

$$\sin(2\pi k) = -\sin(2\pi k)$$

$$2k\pi = n\pi \quad , n = 1, 2, 3$$



$$e^{im_\ell \varphi} = e^{im(\varphi+2\pi)}$$

De kvantiserte energi egenverdiene:

$$k = \frac{n}{2}$$

$$\frac{n^2}{4} = \frac{2mr^2 E_n}{\hbar^2} \Leftrightarrow \underline{\underline{E_n = \frac{\hbar^2}{8mr^2} n^2}}$$

e)

$\frac{\omega}{2\pi}$ = antall δ/s

$$I = e\frac{\omega}{2\pi} \qquad \mu_\ell = e\frac{\omega}{2\pi}\pi r^2 \quad (I \cdot A)$$

$$= \underline{\underline{\frac{e\omega r^2}{2}}}$$

$$L = mvr = m\omega r^2 \qquad \omega r^2 = \frac{L}{m}$$

$$= \underline{\underline{\frac{1}{2} e \frac{L}{m}}}$$

$$\tau = -\vec{\mu}_\ell \times \vec{B}$$

$$E = -\vec{\mu}_\ell \cdot \vec{B} = \frac{eBL}{2m_e} = V \leftarrow \text{Potensialet.}$$

Schrödingers ligning:

$$\frac{-\hbar^2}{2mr^2}\frac{d^2\psi}{d\varphi^2} + \frac{eBL_z}{2m}\psi = E\psi$$

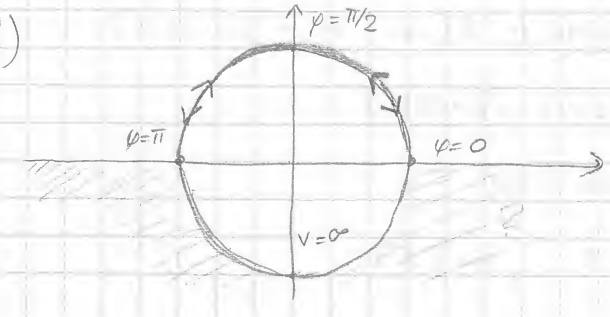
$$\psi(\varphi) = Ce^{ik_n\varphi} + De^{ik_n\varphi}, \quad k_n^2 = \frac{r^2}{\hbar^2}(2mE - eBL_z)$$

Nå blir energi kvantiserte nivåer:

$$k_n = \frac{n}{2} \Leftrightarrow k_n^2 = \frac{n^2}{4}$$

$$\frac{n^2}{4} = \frac{r^2}{\hbar^2}(2mE_n - eBL_z) \Leftrightarrow \underline{\underline{E_n = \frac{\hbar^2}{8mr^2} n^2 + \frac{eBL_z}{2m}}}$$

f)



I: $0 \le \varphi \le \pi$ i $\psi(\varphi) = Fe^{ik\varphi} + Ge^{-ik\varphi}$

$$\psi(0) = 0 \Leftrightarrow F = -G$$

$$\underline{\underline{\psi(\varphi) = 2iF\sin(k\varphi)}}$$

fiem og tilbake

II: $\psi(\Pi) = 0 \Rightarrow \sin(k\pi) = 0$

$k\pi = n\pi \quad , n = 1, 2, \ldots$

$k = n \Rightarrow \frac{2mr^2 E_n}{\hbar^2} = n^2 \Rightarrow \underline{\underline{E_n = \frac{\hbar^2}{2mr^2} n^2}}$

g) Energi egenverdiene for en partikkel i én dimensjon i en vendelig dyp brønn med bredde L er gitt ved

$$\underline{\underline{E_n = \frac{\hbar^2 \pi^2}{2m L^2} n^2}} \quad , n = 1, 2, 3$$

Forskjellen:

3.4

1) $\hat{H}_0 \psi_{n\ell m_\ell}(\vec{r}) = E_n \psi_{n\ell m_\ell}(\vec{r}) \qquad n = 1, 2, 3, \ldots$

2) $\vec{L}^2 \psi_{n\ell m_\ell}(\vec{r}) = \ell(\ell+1)\hbar^2 \psi_{n\ell m_\ell}(\vec{r}) \qquad \ell = 0, 1, 2, \ldots, n-1$

3) $\hat{L}_z \psi_{n\ell m_\ell}(\vec{r}) = m_\ell \hbar \psi_{n\ell m_\ell}(\vec{r}) \qquad m_\ell = -\ell, \ldots, \ell$

4) $\int \psi^*_{n'\ell'm_\ell'}(\vec{r})\, \psi_{n\ell m_\ell}(\vec{r})\, d\vec{r} = \delta_{nn'} \delta_{\ell\ell'} \delta_{m_\ell m_\ell'}$

a)

$\hat{H}_0$ - hele energien til partikkelen

$\vec{L}^2$ - (banespinnet)^2

$\hat{L}_z$ - z-komponenten til banespinnet

b) EGENVERDILIGNINGER

c) Noen fysiske størrelser har skarpe verdier.
Usikkerhetene er null.
Vi får bare en verdi altså egenverdiene til $\psi_{n\ell m_\ell}$

Det er E, $\vec{L}^2$, L_z

d) Ved tiden $t=0$ så gjelder

$$\Psi_{n\ell m_\ell}(\vec{r}) = \Psi(\vec{r},0) \quad \text{, hvor } t=0$$

Vi har T.A.S.L:

$$\hat{\mathcal{H}}_0 \Psi(\vec{r},t) = i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t) \qquad (I)$$

og

$$\Psi(\vec{r},t) = u(\vec{r}) \cdot v(t) \quad \text{settes inn i (I)}$$

slik at:

$$\hat{\mathcal{H}}_0 u(\vec{r}) v(t) = i\hbar \frac{\partial}{\partial t} u(\vec{r}) v(t) \quad \text{, } \hat{\mathcal{H}}_0 \text{ er ikke avhengig av tiden}$$

$$v(t)\, \hat{\mathcal{H}}_0 u(\vec{r}) = u(\vec{r})\, i\hbar \frac{\partial}{\partial t}(v(t)) \quad : u(\vec{r}) v(t)$$

$$\frac{\hat{\mathcal{H}}_0 u(\vec{r})}{u(\vec{r})} = \frac{i\hbar \frac{\partial}{\partial t} V(t)}{V(t)}$$

Disse må være lik en konstant, for alle verdier av x og t.

Konstanten skal vi selke som $\mathcal{E}$, slik at det blir

$$\begin{cases} \hat{\mathcal{H}}_0 u(\vec{r}) = \mathcal{E} u(\vec{r}) = E_n \Psi_{n\ell m_\ell}(\vec{r}) \\ i\hbar \frac{\partial}{\partial t}(v(t)) = \mathcal{E} v(t) \end{cases}$$

$$v(t) = e^{\frac{-i\mathcal{E}t}{\hbar}}$$

$$\Psi(\vec{r},t) = u(r) e^{-i\frac{\mathcal{E}t}{\hbar}}$$

$$\Psi(r,0) = u(r) = \Psi_{n\ell m_\ell}(\vec{r})$$

$$\Psi(\vec{r},t) = u(\vec{r}) v(t) = \Psi_{n\ell m_\ell}(\vec{r}) e^{\frac{-iE_n}{\hbar} t}$$

e)

$$\Psi(r,0) = \Phi(r) = \frac{1}{\sqrt{2\ell+1}} \sum_{m_\ell=-\ell}^{+\ell} \Psi_{n\ell m_\ell}(\vec{r})$$

vise at det er normert:

$$\int \Phi^* \Phi \, d\vec{r} = \int \frac{1}{2\ell+1} \sum_{m_\ell=-1}^{+\ell} \Psi^*_{n\ell m_\ell}(\vec{r}) \sum_{m_\ell=-\ell}^{+\ell} \Psi_{n'\ell' m_{\ell'}}$$

hvor n og ℓ skal være faste.

$$\frac{1}{2l+1}\sum_{ml}\sum_{ml'}\int \psi^*_{nlml}\,\psi_{nlml'}\,d\vec{r} = \frac{1}{2l+1}\sum_{ml}\left(\sum_{ml'}\delta_{ml,ml'}\right)$$

$$= \frac{1}{2l+1}\sum_{ml=-l}^{+l}\left(0+0+0+\dots+1\right) = \frac{1}{2l+1}\sum_{ml=-l}^{l} 1 = \frac{1}{2l+1}\left(2l+1\right) = \underline{\underline{1}}$$

f) $\Psi(\vec{r},t) = \Phi(\vec{r})\,e^{-\frac{iE_n t}{\hbar}}$

$\Psi(\vec{r},t) = u(\vec{r})\,v(t) = u(\vec{r})\,e^{-\frac{i\mathcal{E}}{\hbar}t}$

$u(r) = \Psi(r,0) = \Phi(\vec{r})$ (gu i d)

$$\hat{\mathcal{H}}_0\,u(\vec{r}) = \underline{\hat{\mathcal{H}}_0\,\Phi(\vec{r})} = \mathcal{H}_0\frac{1}{(2l+1)^{1/2}}\sum_{ml=-l}^{+l}\psi_{nlml}(\vec{r})$$

sette inn her

Dette skal virke på vart av disse nlml verdiene.

$$= \frac{1}{\sqrt{2l+1}}\sum_{ml} E_n\,\psi_{nlml}(\vec{r}) = \underline{E_n\,\Phi(\vec{r})}$$

(er den samme for alle nlml)

altså: $\underline{\underline{\mathcal{H}_0\,\Phi(\vec{r}) = E_n\,\Phi(\vec{r})}}$ altså riktig at $\mathcal{E} = E_n$

g) middelverdien: $\langle\mathcal{H}_0\rangle$, $\langle\hat{\vec{L}}^2\rangle$, $\langle\hat{L}_z\rangle$

$$\langle\mathcal{H}_0\rangle = \int \Psi^*(\vec{r},t)\,\hat{\mathcal{H}}_0\,\Psi(\vec{r},t)\,d\vec{r} = \int \Phi^*(\vec{r})\,e^{+\frac{iE_n t}{\hbar}}\,\hat{\mathcal{H}}_0\,\Phi(\vec{r})\,e^{-\frac{iE_n t}{\hbar}}\,d\vec{r}$$

$$= \int e^{\left(\frac{iE_n t}{\hbar}-\frac{iE_n t}{\hbar}\right)}\,\Phi^*\,\hat{\mathcal{H}}_0\,\Phi\,d\vec{r} = \int e^0\,\Phi^*\,\hat{\mathcal{H}}_0\,\Phi\,d\vec{r}$$

$$= \int \Phi^*\,\hat{\mathcal{H}}_0\,\Phi\,d\vec{r} = \int \Phi^*\,E_n\,\Phi\,d\vec{r} = E_n\int \Phi^*\,\Phi\,d\vec{r} = E_n(1) = \underline{\underline{E_n}}$$

Dette er skapp verdi

$$\langle \hat{\vec{L}}^2 \rangle = \int \psi^*_{n\ell m_\ell} \hat{\vec{L}}^2 \psi_{n\ell m_\ell} \, d\vec{r} = \int \Phi^*(r) e^{\frac{iE_n t}{\hbar}} \hat{\vec{L}}^2 \Phi(r) e^{-\frac{iE_n t}{\hbar}} \, d\vec{r}$$

$$= \int e^{\left(\frac{iE_n t}{\hbar} - \frac{iE_n t}{\hbar}\right)} \Phi^*(r) \hat{\vec{L}}^2 \Phi \, dr = \int e^0 \Phi^*(r) \hat{\vec{L}}^2 \Phi(r) \, dr$$

$$= \int \Phi^* \ell(\ell+1)\hbar^2 \Phi \, dr = \ell(\ell+1)\hbar^2 \int \Phi^* \Phi \, dr = \underline{\ell(\ell+1)\hbar^2}$$

$\langle \hat{H}_0 \rangle$ og $\langle \hat{\vec{L}}^2 \rangle$ forventningsverdien er middelverdien, siden det er egenverdi hver gang vi måler får vi alltid $\ell(\ell+1)\hbar^2$ derfor blir dette også middelverdien.

$\langle \hat{L}_z \rangle$ er ikke noe egenverdi, derfor må vi i hvertfall regne den ut.

Fordi $\Phi(r) = \frac{1}{\sqrt{2\ell+1}} \sum_{m_\ell} \psi_{n\ell m_\ell}$

$$\hat{L}_z \Phi(\vec{r}) = \frac{1}{\sqrt{2\ell+1}} \sum_{m_\ell} \hat{L}_z \psi_{n\ell m_\ell} = \frac{\hbar}{\sqrt{2\ell+1}} \sum_{m_\ell} m_\ell \psi_{n\ell m_\ell} \neq \hat{L}_z \Phi(\vec{r}) \; ?$$

Sum av forskjellige egenverdier er ikke lenger egenfunksjon.

Altså:

$$\langle \hat{L}_z \rangle = \int \Psi^*(r,t) \hat{L}_z \Psi(\vec{r},t) \, d\vec{r} = \int \Phi^*(\vec{r}) \hat{L}_z \Phi(\vec{r}) \, d\vec{r} =$$

$$= \frac{1}{2\ell+1} \int \sum_{m_\ell} \psi^*_{n\ell m_\ell} \sum_{m_\ell'} \hat{L}_z \psi_{n'\ell' m_\ell'} \, d\vec{r}$$

$$= \frac{1}{2\ell+1} \sum_{m_\ell=-\ell}^{+\ell} \sum_{m_\ell=-\ell}^{+\ell} \psi^*_{n\ell m_\ell} m_\ell' \hbar \psi_{n\ell m_\ell'} \, d\vec{r}$$

$$= \frac{\hbar}{2\ell+1} \sum_{m_\ell} \sum_{m_\ell'} m_\ell' \int \psi^*_{n\ell m_\ell} \psi_{n\ell m_\ell'} \, d\vec{r}$$

$$= \frac{\hbar}{2\ell+1} \sum_{m_\ell} \left(\sum_{m_\ell'} m_\ell' \delta_{m_\ell, m_\ell'} \right) = \frac{\hbar}{2\ell+1} \sum_{m_\ell} \left(m_\ell' \cdot 1 \right)$$

$$= \frac{\hbar}{2\ell+1} \sum_{m_\ell=-\ell}^{+\ell} m_\ell = \underline{0}$$

h) $\Psi(r,t) = \Phi e^{-\frac{iE_n t}{\hbar}}$ ← tilstandsfunksjon

Beregne fluktuasjonen, $\Delta\hat{H}_0$, $\Delta\hat{\vec{L}}^2$, $\Delta\hat{L}_z$ definert ved $\Delta A = \sqrt{\langle A^2\rangle - \langle A\rangle^2}$

ENERGI $\langle \hat{H}_0^2 \rangle = \int \psi^*_{n\ell m\ell} \hat{H}_0^2 \psi_{n\ell m\ell}\, d\vec{r} = \int \Phi^*(r) e^{+\frac{iE_n t}{\hbar}} \hat{H}_0^2 \Phi(r) e^{-\frac{iE_n t}{\hbar}} d\vec{r}$

$= \int e^{\left(+\frac{iE_n t}{\hbar} - \frac{iE_n t}{\hbar}\right)} \Phi^* \hat{H}_0^2 \Phi\, dr = \int \Phi^* \hat{H}_0^2 \Phi\, dr = \int \Phi^* E_n^2 \Phi\, dr$

$= E_n^2 \int \Phi^* \Phi\, dr = E_n^2 \cdot 1 = \underline{E_n^2}$ $\quad \left[\hat{H}_0^2 \Psi(r,t) = E_n^2 \Psi(r,t)\right]$

$\langle \hat{H}_0 \rangle = \underline{E_n}$ (beregnet tidligere)

Slik at det blir: $\Delta\hat{H}_0 = \sqrt{E_n^2 - (E_n)^2} = \underline{0}$

barespinner: $\langle (\hat{\vec{L}}^2)^2 \rangle = \int \psi^*_{n\ell m\ell} (\hat{\vec{L}}^2)^2 \psi_{n\ell m\ell}\, d\vec{r} = \int \Phi^* (\hat{\vec{L}}^2)^2 \Phi\, d\vec{r}$

$= \int \Phi^* (\ell(\ell+1)\hbar^2)^2 \Phi\, d\vec{r} = (\ell(\ell+1)\hbar^2)^2 \int \Phi^* \Phi\, dr = \underline{\ell(\ell+1)\hbar^2}$

$\langle \hat{\vec{L}}^2 \rangle = \underline{\ell(\ell+1)\hbar^2}$ beregnet tidligere

Slik at det blir: $\Delta\hat{\vec{L}}^2 = \sqrt{(\ell(\ell+1)\hbar^2)^2 - (\ell(\ell+1)\hbar^2)^2} = \underline{0}$

Men: z-komponent: $\langle \hat{\vec{L}}_z^2 \rangle = \int \psi^*_{n\ell m\ell} (\hat{\vec{L}}_z^2) \psi_{n\ell m\ell'}\, d\vec{r} = \int \Phi^* e^{+\frac{iE_n t}{\hbar}} (\hat{\vec{L}}_z^2) \Phi e^{-\frac{iE_n t}{\hbar}} d\vec{r}$

$= \int \Phi^* (\hat{\vec{L}}_z^2) \Phi\, d\vec{r} = \int \left(\frac{1}{\sqrt{2\ell+1}}\right)^2 \sum_{m\ell=-\ell}^{+\ell} \psi^*_{n\ell m\ell}(\vec{r}) (\hat{\vec{L}}_z^2) \sum_{m\ell'=-\ell}^{\ell} \psi_{n\ell m\ell'}\, d\vec{r}$

$= \frac{1}{2\ell+1} \sum_{m\ell=-\ell}^{+\ell} \sum_{m\ell'=-\ell}^{+\ell} \int \psi^*_{n\ell m\ell}(\vec{r}) (\hat{\vec{L}}_z^2) \psi_{n\ell m\ell'}(\vec{r})\, d\vec{r}$

$(n,\ell$ = faste tall) $= \frac{1}{2\ell+1} \sum_{m\ell} \sum_{m\ell'} \hbar^2 m\ell'^2 \int \psi^*_{n\ell m\ell} \psi_{n\ell m\ell'}\, d\vec{r} = \frac{\hbar^2}{2\ell+1} \sum_{m\ell} \left(\sum_{m\ell'} m\ell'^2 \delta_{m\ell m\ell'} \right)$

Vi setter $m\ell = m\ell'$ Rottmann side 115 (2)

$= \frac{\hbar^2}{2\ell+1} \sum_{m\ell=-\ell}^{+\ell} m\ell^2 = \frac{\hbar^2}{2\ell+1} \left(2 \sum_{m\ell=0}^{\ell} m\ell^2\right) = \frac{\hbar^2}{2\ell+1} \left(2 \left(\frac{\ell(\ell+1)(2\ell+1)}{6}\right)\right)$

$= \underline{\frac{\hbar^2 \ell(\ell+1)}{3}}$

Slik at det blir: $\Delta\hat{L}_z = \sqrt{\langle \hat{L}_z^2 \rangle - \langle 0 \rangle} = \underline{\hbar\sqrt{\frac{\ell(\ell+1)}{3}}}$

$$\Phi = \sum_n a_n \psi_{nl} = \left(\frac{1}{2l+1}\right)^{1/2}$$

$$\sum a_n^* = 1 \qquad \int \psi^* \psi \, d\tau = 1$$

i) $P(m_l, \hbar) = ?$ hvor stor er dette.

notat III: $\Phi = C_1\psi_1 + C_2\psi_2 + C_3\psi_3 + \dots$ sum av egenverdier derfor

$C_\nu = \int (\Phi^*) \psi_\nu \, d\tau$. er Φ ikke egenfunksjon

$P_\nu = |C_\nu|^2$ for skjellig verdi for L_z egenverdi

Sannsynlighet:

$$P = \left| \int \Phi^*(r)\, \psi_{n l m_l}(\vec{r})\, d\vec{r} \right|^2$$

$$= \left| \frac{1}{\sqrt{2l+1}} \int \sum_{m_l'=-l}^{+l} \psi^*_{n l m_l'} \psi_{n l m_l} \, d\tau \right|^2 \quad \text{basert me}$$

$$= \frac{1}{2l+1} \left| \sum_{m_l'} \int \psi^*_{n l m_l'} \psi_{n l m_l} \, d\vec{\tau} \right|^2 \quad \text{over rommet}$$

$$= \frac{1}{2l+1} \left| \sum_{m_l'} \delta_{m_l m_l'} \right|^2 = \underline{\underline{\frac{1}{2l+1}}}$$

j) $\psi(r,t) = \Phi(r) e^{-\frac{iEnt}{\hbar}} = \psi_{n l m_l}(r) e^{-\frac{iEnt}{\hbar}}$

$$P = \left| \int \Phi^*(r) e^{+\frac{iEnt}{\hbar}} \psi_{n l m_l} e^{-\frac{iEnt}{\hbar}} \right|^2 = \left| \int e^{0} \Phi^*(r) \psi_{n l m_l} \right|^2$$

$$= \frac{1}{2l+1} \left| \sum \int \psi^*_{n l m_l} \psi_{n l m_l} \, d\tau \right|^2 = \frac{1}{2l+1} \left| \sum \delta_{m_l m_l'} \right|^2 = \underline{\underline{\frac{1}{2l+1}}}$$

altså uavhengig av tiden.

k) $\hat{\mathcal{H}}\psi = \left(\hat{\mathcal{H}}_0 + \frac{eB}{2m} \hat{L}_z \right) \psi$

$$E = E_n + \frac{eB}{2m} m_l \hbar$$

e) $\hat{H}\Phi = (\hat{H}_0 + \frac{eB}{2m}\hat{L}_z)\Phi$

$= E_n \Phi + \frac{eB\hbar}{2m} \hat{L}_z \frac{1}{\sqrt{2l+1}} \sum_{m_l=-l}^{+l} \psi_{n l m_l}(\vec{r})$

, $L_z \sum_{m_l=-l}^{+l} \psi_{n l m_l} = -l\hbar\psi - (l+1)\hbar\psi_+ \cdots + l\hbar\psi$

Skulle hatt: $\phi\psi = \text{tall}\cdot\psi$

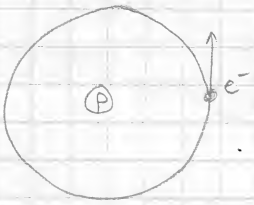
men L_z gir forskjellig tall dersom vi ikke har (en energi egentilstand)

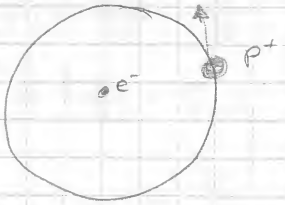
$L_z \phi(\vec{r}) = \hat{L}_z \frac{1}{\sqrt{2l+1}} \sum_{m_l=-l}^{+l} \psi_{n l m_l}$

$= \frac{1}{\sqrt{2l+1}} \sum_{m_l=-l}^{+l} m_l \hbar \psi_{n,l,m_l}$

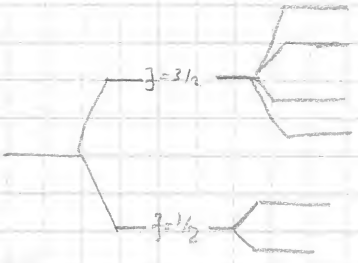
m_j → forteller degenerasjonen

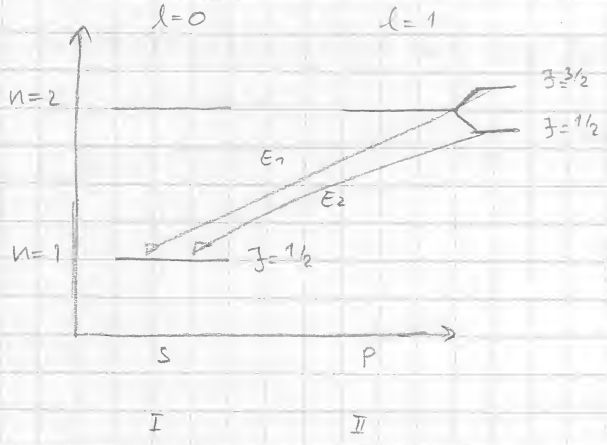
3.6





$L = \sqrt{l(l+1)}\,\hbar$ $\quad L_z = m_l \hbar$, $m_l = \pm l$ $\quad \vec{J} = \vec{L} + \vec{S}$





$|l-s| \le j \le l+s$

I: $\left|0-\frac{1}{2}\right| \le j \le 0+\frac{1}{2}$

$\frac{1}{2} \le j \le \frac{1}{2}$

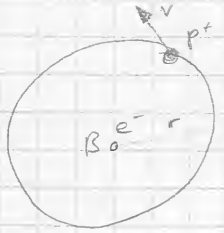
II: $\left|1-\frac{1}{2}\right| < j \le 1+\frac{1}{2}$

$\frac{1}{2} < j < 3/2$

To energi overganger

$$\left.\begin{aligned} E_1 &= h\nu_1 = h\frac{c}{\lambda_1} \\ E_2 &= h\nu_1 = h\frac{c}{\lambda_1} \end{aligned}\right\} \quad \Delta\lambda = \lambda_1 - \lambda_2 = \underline{5{,}3\cdot 10^{-13}\,m}$$

b)



$$\vec{B} = E \times \frac{\vec{V}}{c^2} \quad , \quad \vec{E} = \frac{e}{4\pi\epsilon_0 r^2}\frac{\vec{r}}{r}$$

$$E_{\mu B} = \vec{\mu_B}\cdot\vec{B} \quad , \quad \vec{\mu_B} = \frac{e}{m}\vec{S}$$

$$\vec{B} = \frac{e}{4\pi\epsilon_0 r^2 c^2}\frac{\vec{r}}{r}\times\vec{V} = \underline{\frac{e}{4\pi\epsilon_0 r^3 c^2 m}\vec{L}}$$

p.g.a. det er elektron i sentrum, protonet må gå rundt den for å få indusert B-felt på e^-.

$$\hat{H}_{LS} = E_{\mu B} = \frac{e^2}{4\pi\epsilon_0 m^2 c^2 r^3}\hat{\vec{L}}\cdot\hat{\vec{S}} = A\vec{L}\cdot\vec{S}$$

hvor $r = a_0 n^2$, $n=2$ $\quad \underline{\hbar^2 A = 2\cdot 10^{-5}\,eV}$

3.8

a) Skal bestemme hvor mange elektroner det kan være i

1s; 2s, 2p; 3s, 3p, 3d; ----; nl = n0, nl = n1, ---, nl = n, n-1, tilstanden

Vi benytter av formelen $NL^{2(2l+1)}$ til å bestemme antall m_l-er $\underset{m_s=\pm\frac{1}{2}}{2l+1}(2(2l+1))$

dette m.a.o L-kvantetallet bestemmer hvor mange elektroner som kan eksistere i hver tilstand.

for s tilstanden er $l=0$ => antall elektroner 2

p	$l=1$	6
d	$l=2$	10
f	$l=3$	14

1s - 2 elektroner, 2s 2p - 8 elektroner, 3s 3p 3d - 18 elektroner

4s 4p 4d 4f - 32 elektroner (BRUK: $2\sum_{l=0}^{n=1} 2l+1 = 2n^2$)

Pauli prinsippet - INGEN ELEKTRONER KAN VÆRE I SAMME KVANTE TILSTAND.

b) De bundne tilstandene i hydrogen atomet er bare avhengig av kvantetallet n. Alle nivåene med forskjellig l har samme energi uavhengig av verdier til m_l, er en konsekvens av kulesymmetrien til systemet. Kulesymmetrien har ingen spesiell retning i rommet. Derfor kan ikke energien være avhengig av L_z.

P.G.A. symmetrisk ødelagt, forskjell potensiell energi, coulomb felt, osv, frastøting av elektroner.

c)

He har 2 elektroner, Be 4 elektroner og Ne 10 elektroner

Grunntilstanden for disse betegnes med symbolet $^1S_0$ ved bruk av formelen $^{2S+1}L_J$

For å få symbolet $^1S_0$ for alle tre, må alle tre atomene ha $s=0$, $L=0$ og $j=0$ i grunntilstanden.

He:

$$\left[\begin{array}{l} \vec{L}_1 + \vec{L}_2 = 0 \\ \vec{S} = s_1 + s_2 = 0 \end{array}\right] \quad \boxed{S}_0$$

$n = 1 \rightarrow L_1 = L_2 = 0 \quad L = 0$

$S = s_1 + s_2 = 1/2 - 1/2 = 0$

$J = |L \pm S| = 0$

K $\boxed{\uparrow\downarrow}$ S

Be:

L $\boxed{\uparrow\downarrow}$

K $\boxed{\uparrow\downarrow}$

S

$n = 1 \Rightarrow L = L_1 + L_2 + L_3 + L_4 = 0$

$S = s_1 + s_2 + s_3 + s_4 = 1/2 - 1/2 + 1/2 - 1/2 = 0$ under skall fylt

$J = |L \pm S| = 0$

$[n = 1, \ l = 0, \ m_l = 0 \quad s_1 = +\frac{1}{2}\hbar \quad s_2 = -\frac{1}{2}\hbar]$

Ne:

$n = 1 \Rightarrow L = L_1 + L_2 + \dots + L_{10} = 0$

$S = s_1 + s_2 + \dots + s_{10} = 0$

$J = |L \pm S| = 0$

L $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow\downarrow}$

K $\boxed{\uparrow\downarrow}$

S P

$$\left[\begin{array}{ll} n=1, & l=0, \ m_l=0, \ m_s = \pm 1/2\hbar \\ n=2, & l=0, \ m_l=0, \ m_s = \pm 1/2\hbar \\ & \cancel{l=1} \quad \text{elektroner havner ikke her} \end{array}\right]$$

d) Na: 11 elektroner Grunntilstand $^2S_{1/2}$.

Na:

M $\boxed{\uparrow}$

L $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow\downarrow}$ $\boxed{\uparrow\downarrow}$

K $\boxed{\uparrow\downarrow}$

S P

$n=1$

$n=2$

$n=3, \ l=0, \ m_l=0, \ m_s = \pm 1/2\hbar$

Både K og L skallet er fullt det betyr at totale L og S for disse elektroner er 0. Dermed er spinn kvantetallet bestemt av valens elektronets kvantetall $:\Rightarrow$ $S = 1/2$ $L = 0$ får da $^2S_{1/2}$.

(Se regel 1, hefte).

e) De to laveste eksisterte tilstandene i Na-atomet er $^2P_{1/2}$ og $^2P_{3/2}$

Her er valenselektronet eksitert til P-atomet altså $l=1$

dvs. $J = |L \pm S| = |1 \pm 1/2| = 1/2$ og $3/2$

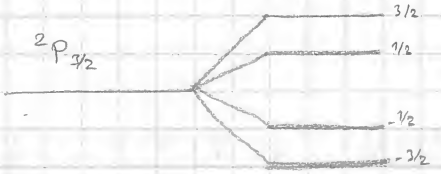
Får da $^{2\cdot 1/2+1}P_{1/2}$ og $^{2\cdot 1/2+1}P_{3/2}$ altså $^2P_{1/2}$ og $^2P_{3/2}$

f) Årsaken til energiforskjellen mellom disse eksisterte tilstandene er at kvantetallet J blir forskjellig og vi får to tilstander.

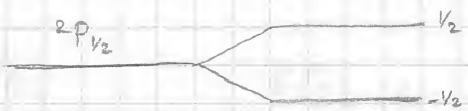
LS-kobling $\vec{L}\cdot\vec{S} \neq 0$.

Degenerasjonsgraden er bestemt av kvantetallet m_j. Dette er definert ved $J: m_j = -J, -J+1, \ldots, J$

Antallet forskjellige m_j verdier tilsvarer antall deg. nivåer. Bestemt av $2J+1$



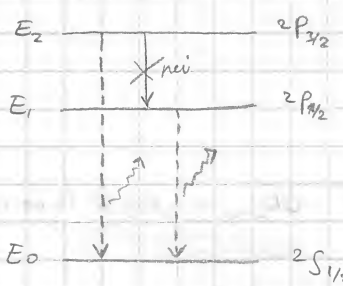
4 forskjellige m_j verdier. Degenerasjonsgrad 4



2 forskjellige m_j verdier. Degenerasjon grad 2.

OR: $2J+1 = \begin{matrix} 2\cdot 1/2 + 1 = 2 \\ 2\cdot 3/2 + 1 = 4 \end{matrix} = 6$

g) Na-atomet



Kravene må oppfylles $\Delta j = 0, \pm 1$ } VIKTIG
$\Delta L = \pm 1$ }

Altså ifølge konvensjonen, så bør det gå slik.

$^2P_{1/2} \longrightarrow {}^2S_{1/2}$ og $^2P_{3/2} \longrightarrow {}^2S_{1/2}$

Skal finne oppløsningen for spektrometeret slik at bølgelengdene kan skilles.

$E_1 = E(^2P_{1/2}) = 2{,}1022\,eV$

$E_2 = E(^2P_{3/2}) = 2{,}1044\,eV$

$$\Rightarrow \Delta\lambda = \lambda_1 - \lambda_2 = \frac{hc}{E_1} - \frac{hc}{E_2} = \frac{hc(E_2 - E_1)}{E_2 E_1} = \frac{12400\,eV\text{Å}\cdot 0{,}0022\,eV}{(6{,}1022 \cdot 2{,}1044)\,eV^2} = \underline{6{,}2\text{Å}}$$

h) Operatoren for det magnetiske dipolmomentet er gitt ved

$\vec{M} = \frac{-e}{2m} g \vec{J}$ hvor $\vec{J}$ er operatoren for det totale spinnet

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \qquad {}^{2S+1}L_J$$

Regner ut g:

- $^2S_{1/2} \Rightarrow J = 1/2, \; L = 0, \; S = 1/2$

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 0(0+1)}{2(1/2(1/2+1))} = 1 + \frac{\frac{3}{4}+\frac{3}{4}}{3/2} = \underline{2} \quad \text{q.e.d}$$

- $^2P_{3/2} \Rightarrow J = 3/2, \; L = 1, \; S = 1/2$

$$g = 1 + \frac{\frac{3}{2}(\frac{3}{2}+1) + \frac{1}{2}(\frac{1}{2}+1) - 1(1+1)}{2\,\frac{3}{2}(\frac{3}{2}+1)} = 1 + \frac{\frac{15}{4}+\frac{3}{4}-2}{15/2} = \frac{4}{3} \text{ q.e.d}$$

- $^2P_{1/2} \Rightarrow J = 1/2, \; L = 1, \; S = 1/2$

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 1(1+1)}{2\,1/2(1/2+1)} = 1 + \frac{\frac{3}{4}+\frac{3}{4}-2}{3/2} = \frac{2}{3} \text{ q.e.d.}$$

i) $\hat{H}' = -\hat{\vec{M}} \cdot \vec{B} = -\left(-\frac{e}{2m} g \vec{j} \cdot \vec{B}\right) = \frac{e}{2m} g \vec{J} \cdot \vec{B}$ $\qquad |\vec{B}| = B_z$, $\vec{J} = \vec{J}_z$ og $J_z = m_j \hbar$

$$= \frac{e\hbar}{2m} g m_j B = \mu_B \cdot g \cdot m_j \cdot B$$

$$\boxed{\Delta E = \mu_B \cdot g \cdot m_j \cdot \vec{B}}$$

Vi har regnet ut energinivåer (se tilbake)

$^2S_{1/2} \longrightarrow j = 1/2, \; m_j = \pm 1/2, \; g = 2$

$$\Delta E = \mu_B \cdot 2 (\pm 1/2) B$$

$\Delta E_{1/2} = \mu_B \cdot B$ (når $m_j = +1/2$) $\qquad \vee \qquad \Delta E_{-1/2} = -\mu_B B$ (når $m_j = -1/2$)

$^2P_{1/2} \longrightarrow j = 1/2, \; m_j = \pm 1/2, \; g = 2/3$

$$\Delta E = \mu_B \cdot \frac{2}{3} (\pm 1/2) B$$

$\Delta E = \frac{1}{3} \mu_B B$ $(m_j = 1/2)$ $\qquad \vee \qquad \Delta E = -\frac{1}{3} \mu_B B$ $(m_j = -1/2)$

$^2P_{3/2}$ $\quad J = 3/2, \quad m_j = \pm 3/2 \quad g = 4/3 \quad (\Delta j = \pm 1)$

(I) $\Delta E = \mu_B \left(\frac{4}{3}\left(\pm \frac{3}{2}\right)\right) B$

$\Delta E = \mu_B B(\pm 2)$

$\Delta E = 2\mu_B B \quad (m_j = 3/2)$ $\qquad$ $\Delta E = -2\mu_B B \quad (m_j = -3/2)$

(II) $\Delta E = \mu_B \left(\frac{4}{3}\right)\left(\pm \frac{1}{2}\right) B$

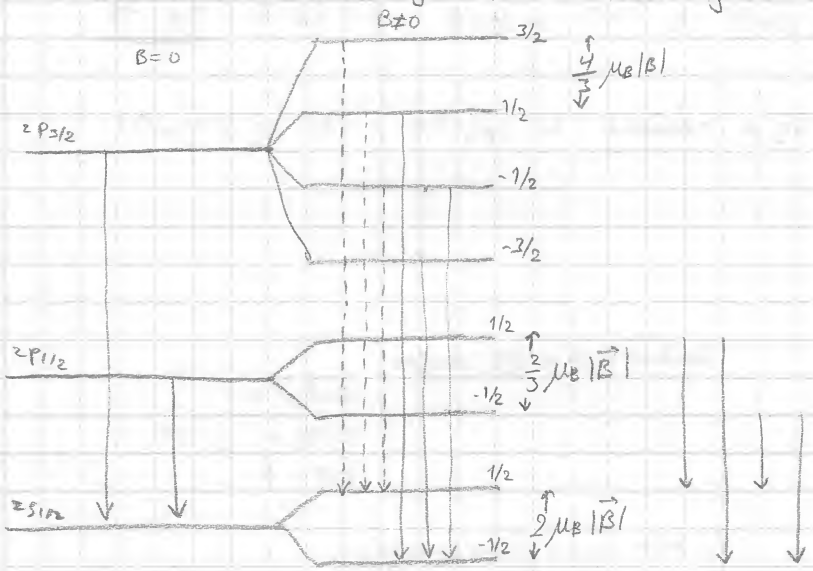
$= \frac{2}{3}\mu_B \cdot B \quad (m_j = 1/2)$ $\qquad$ $\Delta E = -\frac{2}{3}\mu_B \cdot B \quad (m_j = -1/2)$

j) Utvalgsregler:

$\Delta j = 0, \pm 1 \qquad \Delta m_j = 0, \pm 1 \qquad \Delta L = \pm 1$

↓

betyr at vi ikke kan ha overgang mellom $^2P_{3/2} \rightarrow {}^2P_{1/2}$



3.11 V92#2

a) Li-atom

∘ $^2D_{5/2}$ kommer fra formelen $^{2S+1}L_j$ der $j = L+S$

det betyr at kvantetallene er følgende, $S = 1/2$, $L = 2$, $J = 5/2$

mj verdier forteller oss om degenerasjonsgraden. Antall mj verdier er lik degenerasjonsgraden $(2j+1)$

Får da $m_j = \pm\frac{5}{2}, \pm\frac{3}{2}, \pm\frac{1}{2} \Rightarrow 6$ eller deg 6 = 2j+1

∘ 3s nivået er $l = 0$, $s = 1/2$ får da $J = |L \pm S| = \frac{1}{2}$

da er $m_j = \pm 1/2 \Rightarrow 2$, deg 2 = 2j+1

b)

Spinn-bane koblingen $\vec{S} \cdot \vec{L}$

p-state $\rightarrow l = 1 \rightarrow s = 1/2$ summeres opp.

s nivået er $l = 0$
p nivået er $l = 1$
d — " — $l = 2$

Herfra formelen $j = |l \pm s|$ der $s = 1/2$ ser at for s nivået er $j = \underline{1/2}$

mens for p nivået har j to verdier nemlig $3/2$ og $1/2$ p.g.a

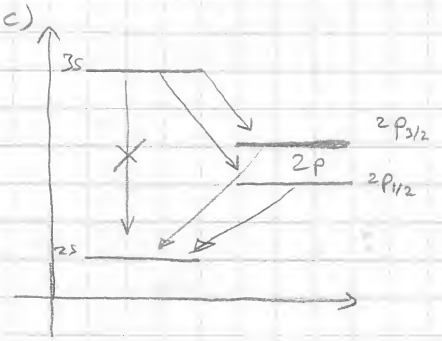
P-state $\rightarrow l = 1$ og $s = 1/2$ $\quad j = 1 \pm 1/2 = \underline{3/2, 1/2}$

mens for d nivået har j to verdier nemlig $5/2$ og $3/2$ p.g.a

D-state $\rightarrow l = 2$ og $s = 1/2$ $\quad j = 2 \pm 1/2 = \underline{5/2, 3/2}$

Dette er grunnen til at P og d nivåene er splittet i to og s nivået splittet i 1 nivået.

c)



Bruker utvalgte regler

$\Delta l = \pm 1$
$\Delta m_l = 0, \pm 1$
$\Delta s = 0$

Vi kan ha 4 fotonenergien.

d)

$3s \longrightarrow {}^2P_{3/2}$	$E = \frac{hc}{\lambda}$	$\Delta E = 1{,}5279\,eV$	$\lambda = 8{,}1147 \cdot 10^{-7}\,m$
$3s \longrightarrow {}^2P_{1/2}$		$\Delta E = 1{,}5281\,eV$	$\lambda = 8{,}1136 \cdot 10^{-7}\,m$
$^2P_{3/2} \longrightarrow 2s$		$\Delta E = 1{,}8511\,eV$	$\lambda = 6{,}6979 \cdot 10^{-7}\,m$
$^2P_{1/2} \longrightarrow 2s$		$\Delta E = 1{,}8509\,eV$	$\lambda = 6{,}6987 \cdot 10^{-7}\,m$

3.12 V. 93 #3

Energi egenverdiene for et elektron i et hydrogén-atom er da gitt ved

$$\mathcal{H}_0 \psi_{n\ell m_\ell} = -E_0 \frac{1}{n^2} \psi_{n\ell m_\ell} = \frac{-13{,}6}{n^2} \psi_{n\ell m_\ell}$$

a) Hamilton-operatoren $\hat{H}_0$ er da:

$$\hat{\mathcal{H}}_0 = \frac{-\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

hvor $n \geq 1$ $(n = 1, 2, 3, \ldots)$ & $l \leq n-1$, $(l = 0, 1, 2, 3, 4, \ldots, n-1)$

$|m_\ell| \leq l$, $(m_\ell = -l, -l+1, \ldots, -1, 0, 1, \ldots, l-1, l)$

b) Kvantetallet m_ℓ tilfredstiller egenverdi-ligningen

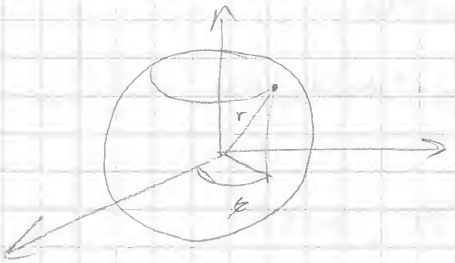
$$\left(\frac{d^2}{d\varphi^2} + m_\ell^2\right)\Phi(\varphi) = 0$$

Løsningen er:

$$\Phi(\varphi) = A\sin(m_\ell\varphi) + B\cos(m_\ell\varphi)$$

også:

$$\Phi(\varphi) = Ce^{im_\ell\varphi}$$



$$\Phi(\varphi) = Ce^{im_\ell\varphi} = Ce^{im(\varphi+2\pi)}$$

det impliserer at $m_\ell = 0, \pm 1, \pm 2, \ldots$ heltall

c) Bølgefunksjonen for s-tilstanden med $n=2$ er gitt ved

$$\psi_{200} = \sqrt{\frac{1}{8\pi a_0^3}}\left(1 - \frac{\rho}{2}\right)e^{(-\rho/2)}, \quad \rho = \frac{r}{a_0}$$

Normeringen gir:

$\rho^2 = \frac{r^2}{a_0^2}$

$2\rho\, d\rho\, a_0^2 = 2r\,dr$

$$N = \int \psi_{200}^* \psi_{200}\, r^2 \sin\theta\, d\theta\, d\varphi\, d\, dr = \int \psi_{200}^* \psi_{200}\, r\sin\theta\, d\theta\, d\varphi\, r\,dr =$$

$$= \int \psi_{200}^* \psi_{200}\, \rho a_0 \sin\theta\, d\theta\, d\varphi\, \rho a_0^2\, d\rho = \int \psi_{200}^* \psi_{200}\, \rho^2 a_0^3 \sin\theta\, d\varphi\, d\rho$$

$$= \int_0^\pi \int_0^{2\pi} \int_0^\infty \psi_{200}^* \psi_{200}\, \rho^2 a_0^3 \sin\theta\, d\varphi\, d\rho = 4\pi a_0^3 \int \psi_{200}^* \psi_{200}\, \rho^2\, d\rho$$

ingen komplekse størrelser $\Rightarrow$ ta rett og slett uttrykket i 2. potens.

$$= 4\pi a_0^3 \int_0^\infty \left(\sqrt{\frac{1}{8\pi a_0^3}}\right)^2 \left(e^{(-\frac{r}{2a_0})} - \frac{r}{2a_0} e^{(-r/2a_0)}\right)\left(e^{(-r/2a_0)} - \frac{r}{2a_0} e^{(-r/2a_0)}\right) d\rho$$

$$= 4\pi a_0^3 \cdot \frac{1}{8\pi a_0^3} \int_0^\infty \rho^2 \left(e^{-\frac{r}{a_0}} - \frac{2 \cdot r}{2a_0} e^{(-r/a_0)} + \frac{r^2}{4a_0^2} e^{(-r/a_0)}\right) d\rho = \frac{1}{2}\int_0^\infty \rho^2 \left(1 - \rho + \frac{1}{4}\rho^2\right) e^{-\rho} d\rho$$

NB!

$$= \frac{1}{2}\int_0^\infty \left(\rho^2 - \rho^3 + \frac{1}{4}\rho^4\right) e^{-\rho} d\rho = \frac{1}{2}\Big[$$

$$= 1 - 3 + 3 = \underline{\underline{1}}$$

$$\langle r \rangle = \frac{a_0}{2}\int_0^\infty \left(\rho^3 - \rho^4 + 1/4\,\rho^5\right) e^{-\rho} d\rho = \frac{a_0}{2}(6 - 24 + 30) = 6a_0$$

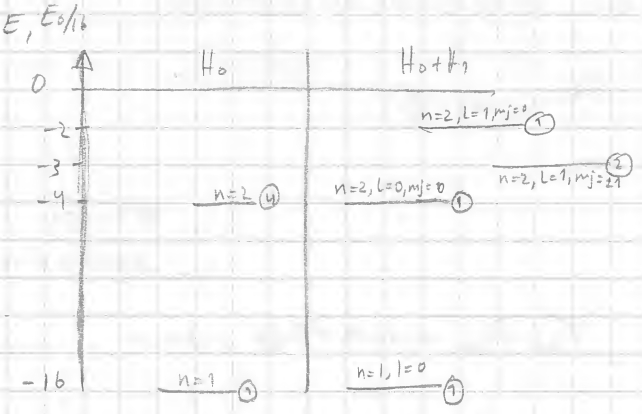
d)

$\hat{H}_{tot} = \hat{H}_0 + \hat{H}_1$, hvor $\hat{H}_1 = \frac{E_0}{16\hbar^2}(\hat{L}_x^2 + \hat{L}_y^2)$ hvor $\hat{L}_x^2$ og $\hat{L}_y^2$ ikke er kvantiserte størrelser, derfor omformer til

$$\hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 = \hat{L}_{tot}^2 - \hat{L}_z^2$$

altså: $\hat{H}_{tot}\psi(x) = (\hat{H}_0 + \hat{H}_1)\psi(x) = \left(-\frac{E_0}{n^2} + \frac{E_0}{16\hbar^2}\left(l(l+1)\hbar^2 - (ml\hbar)\right)\right)\psi(x)$

fordi vi har at $\hat{L}\psi = l(l+1)\hbar^2\psi$ og $-\hat{L}_z^2\psi = -(ml\hbar)^2\psi$



÷ energi → bindingsenergi

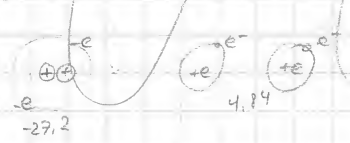
4.1

a) $P+P+e+e$ setter langt fra hverandre, har da $E=0$ når to hydrogen setter sammen: $H+H: 2\cdot(-13,0\,eV) = \underline{-27,6\,eV}$

$H_2^+ + e: (-13,6 - 2,65)\,eV = \underline{-16,25\,eV}$

fremdeles -16,25eV selv om vi setter inn i uendelig, uendret svar

b)

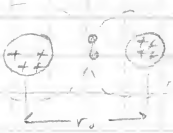
c) 

H_2: (-27,2 - 4,48) eV

(27,2 - 16,25) eV

4.2

Det finn molekyler som binder sammen!



a) $\hat{H}\Psi = E\Psi$ ← kan anvendt på modeller, f.eks. Morse potensial

Morse potensialet er gitt ved:

$$V(r) = D\{e^{-2a(r-r_0)} - 2e^{-a(r-r_0)}\}$$

Skisser dette:

$$V'(r) = D\{-2ae^{-2a(r-r_0)} + 2ae^{-a(r-r_0)}\} = 0$$

$D=0$ eller:

$$-2ae^{-2a(r-r_0)} + 2ae^{-a(r-r_0)} = 0$$

$$-2ae^{-2a(r-r_0)} = -2ae^{-a(r-r_0)}$$

$$e^{-2a(r-r_0)} = e^{-a(r-r_0)}$$

$$\ln e^{-2a(r-r_0)} = \ln e^{-a(r-r_0)}$$

$$-2ar + 2ar_0 = -ar + ar_0$$

$$-ar = ar_0 \iff \underline{r = r_0}$$

Dette er et ekstremal punkt:

må sjekke om dette er min eller maks

dobbelt derivert løst:

$$V'(r) = D\{-2ae^{-2a(r-r_0)} + 2ae^{-a(r-r_0)}\}$$

$$V''(r) = D\{-4a^2e^{-2a(r-r_0)} - 2a^2e^{a(r-r_0)}\}$$

$D=0$ eller:

$$4a^2e^{-2a(r-r_0)} = 2a^2e^{a(r-r_0)}$$

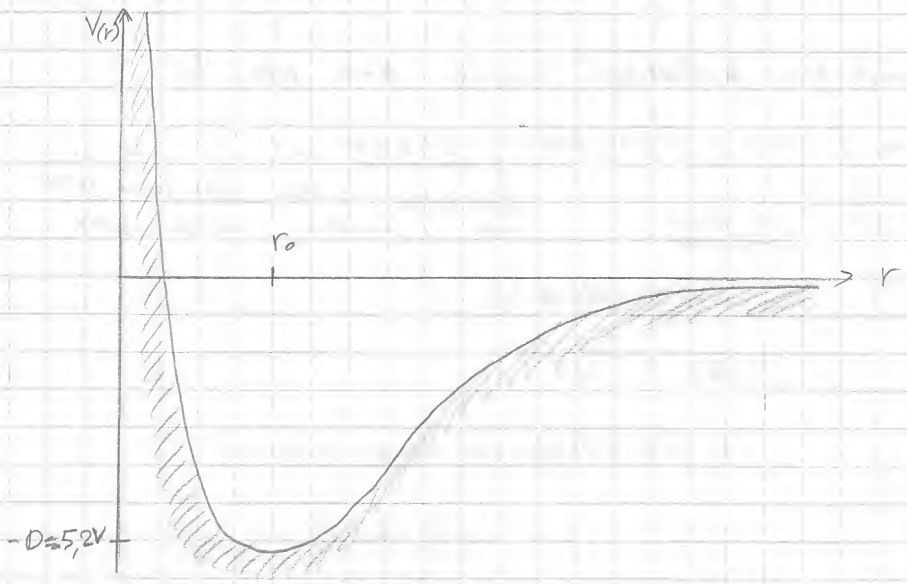
$r=r_0 \Rightarrow$

$$4a^2e^0 = 2a^2e^0$$

$$4a^2 = 2a^2$$

$$4a^2 - 2a^2 = 0$$

$\underline{2a^2 > 0}$ altså minimum, skissert foran:



b) Det trengs $-5{,}2\,eV$ for å spalte et molekyl, fordi ved uendelig r, $\Rightarrow$ atomene separeres ved uendelig avstand, så vekselvirkes det ikke hverandre, derfor.

c) Vi rekker utvikler rundt likevektsavstanden R_0:

$$V(r_0) = V(r_0) + \underbrace{(r-r_0)\frac{dV}{dr}}_{0} + \frac{(r-r_0)^2}{2!}\frac{d^2V}{dr^2} + \frac{(r-r_0)^3}{3!}\frac{d^3V}{dr^3} + \dots$$

$$V(r_0) \approx \frac{1}{2}\frac{d^2V}{dr^2}(r-r_0)^2 + V(r_0)$$, det betyr at $\underline{k = \frac{d^2V}{dr^2}}$

konstanten k er da:

$$V(r) = D\left\{e^{-2a(r-r_0)} - 2e^{-a(r-r_0)}\right\} \qquad r - r_0 = x$$

$$V''(r) = D\left\{4a^2e^{(r-r_0)} - 2a^2e^{-a(r-r_0)}\right\} \qquad V(r) = V(r_0+x)$$

$$V''(r=R_0) = D\left\{4a^2 - 2a^2\right\} = \underline{\underline{D2a^2}}$$

d) sirkelfrekvensen er gitt ved $\omega_v = \sqrt{k/\mu}$ $\qquad \mu = \frac{m_1 \cdot m_2}{m_1 + m_2} = \frac{m^2}{2m} = \underline{\frac{1}{2}m}$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{1}{2\pi}\sqrt{\frac{D2a^2}{\frac{1}{2}m}} = \frac{1}{2\pi}\frac{2a}{1}\sqrt{\frac{D}{m}} = \underline{\frac{a}{\pi}\sqrt{\frac{D}{m}}} = \frac{-27}{\pi \cdot 10^{-9}}\sqrt{\frac{5{,}2\,eV \cdot 1{,}6 \cdot 10^{-19}}{16 \cdot 1{,}66 \cdot 10^{-27}}} = 1{,}22 \cdot 10^{6}$$

$$= \underline{1{,}22\,MHz}$$

48 MHz

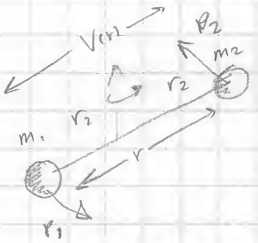
$\underline{4{,}8 \cdot 10^{13}\,Hz}$?

e) $E_{n=0} = (n+1/2)\frac{h}{2\pi}\sqrt{\frac{k}{\mu}} = (n+1/2)\hbar\omega = (n+1/2)\frac{\hbar}{2a}\sqrt{\frac{D}{m}}$

$= \frac{1}{2}\hbar 2a\sqrt{\frac{5{,}2\,eV}{m}} = \frac{1}{2}\cdot 6{,}58\cdot10^{-16}\cdot 2\cdot\frac{27}{10^{-9}}\cdot\sqrt{\frac{5{,}2\,eV}{16\cdot 931{,}5\,MeV/c^2}} = \underline{0{,}1\,eV}$

$E_{n=0} = 5{,}2eV - 0{,}1eV = \underline{5{,}1eV}$ grunntilstanden

4.3



a) Skulle vise at vi får dette når vi har $m_1 r_1 = m_2 r_2$

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$$

Bevis: $|p_1| = |p_2|$

$$\mathcal{H} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(r) = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} + V(r)$$

$$= \frac{p^2}{2}\frac{m_1+m_2}{m_1 m_2} = \frac{p^2}{2\mu} \quad QED$$

b) $$\hat{\mathcal{L}} = \frac{-\hbar^2}{2\mu}\cdot\nabla^2 + V(r)$$

$$= \frac{-\hbar^2}{2\mu}\left(\underbrace{\frac{1}{r_0^2}\frac{\partial}{\partial r}r_0^2\frac{\partial}{\partial r}}_{\text{konstant}} - \frac{1}{\hbar^2 r_0^2}\hat{L}^2\right) + \underbrace{V(r_0)}_{\text{konstant}}$$

$$= \frac{-\hbar^2}{2\mu}\left(-\frac{\hat{L}^2}{\hbar^2 r_0^2}\right) + \text{konstantene}$$

$$= \underline{\frac{\hat{L}^2}{2\mu r_0^2}}$$

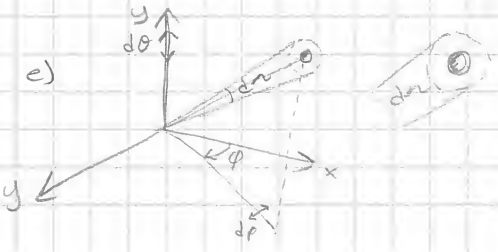
c) Det er rotasjons bevegelse den utfører.

d) $\hat{L}^2\Psi_E = \hbar^2 l(l+1)\Psi_E$ $\qquad \hat{H}\Psi_E = \frac{\hat{L}^2}{2\mu r_0^2}\Psi_E = \frac{\hbar^2 l(l+1)}{2\mu r_0^2}\Psi_E$

kap. 8.3 $[\hat{L}_z, \hat{L}^2] = 0$ og $[\hat{L}_z, \hat{H}] = 0$

L_z kan også ha skarpe verdier. Ser bort fra konstanten.

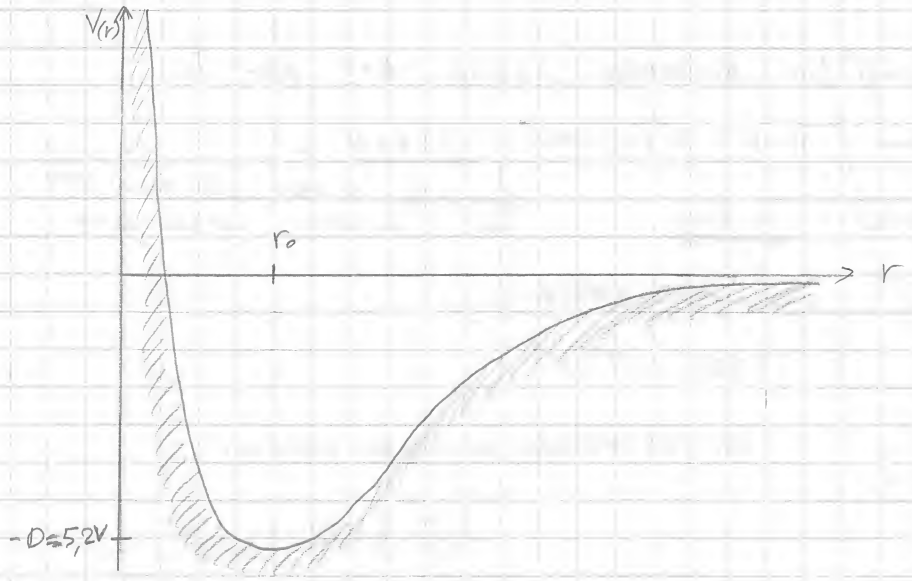
e)



$|\Psi_E|^2 \underbrace{\sin\theta\, d\theta d\varphi}_{}$ = angular probability

$|\Psi_E|^2 dr$ (dr romvinkel)

$\Psi_E = \Theta(\theta)\Phi(\varphi)$



b) Det trengs -5,2 eV for å spalte et molekyl, fordi ved uendelig r, så atomene separeres ved uendelig avstand, så vekselvirker det ikke hverandre. Derfor.

c) Vi tekker utviklen rundt likevektsavstanden R_0:

$$V(r_0) = V(r_0) + (r-r_0)\frac{dV}{dr} + \frac{(r-r_0)^2}{2!}\frac{d^2V}{dr^2} + \frac{(r-r_0)^3}{3!}\frac{d^3V}{dr^3} + \cdots$$

$\frac{dV}{dr} = 0$

$$V(r_0) \approx \frac{1}{2}\frac{d^2V}{dr^2}(r-r_0)^2 + V(r_0), \text{ det betyr at } \underline{k = \frac{d^2V}{dr^2}}$$

konstanten k er da:

$$V(r) = D\{e^{-2a(r-r_0)} - 2e^{-a(r-r_0)}\} \qquad r - r_0 = x$$

$$V''(r) = D\{4a^2e^{(r-r_0)} - 2a^2e^{-a(r-r_0)}\} \qquad V(r) = V(r_0+x)$$

$$V''(r=R_0) = D\{4a^2 - 2a^2\} = \underline{D2a^2}$$

d) Sirkelfrekvensen er gitt ved $\omega_0 = \sqrt{k/\mu}$ $\qquad \mu = \frac{m_1 \cdot m_2}{m_1 + m_2} = \frac{m^2}{2m} = \underline{\frac{1}{2}m}$

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{1}{2\pi}\sqrt{\frac{D2a^2}{\frac{1}{2}m}} = \frac{1}{2\pi}\frac{2a}{1}\sqrt{\frac{D}{m}} = \underline{\frac{a}{\pi}\sqrt{\frac{D}{m}}} = \frac{-27}{\pi \cdot 10^{-9}}\sqrt{\frac{5{,}2\,eV \cdot 1{,}6 \cdot 10^{-19}}{16 \cdot 1{,}66 \cdot 10^{-27}}} = 1{,}22 \cdot 10^{6}$$

$= \frac{1{,}22\,MHz}{48\,MHz}$

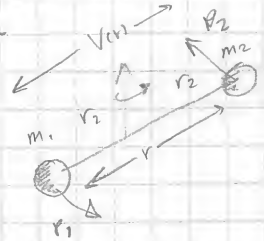
$\underline{4{,}8 \cdot 10^{13}\,Hz}$?

e) $E_{n=0} = (n+1/2)\frac{h}{2\pi}\sqrt{\frac{k}{\mu}} = (n+1/2)\hbar\omega = (n+1/2)\frac{\hbar}{2a}\sqrt{\frac{D}{m}}$

$= \frac{1}{2}\hbar 2a\sqrt{\frac{5{,}2\text{eV}}{m}} = \frac{1}{2}\cdot 6{,}58\cdot10^{-16}\cdot 2\cdot\frac{27}{10^{-9}}\cdot\sqrt{\frac{5{,}2\text{eV}}{16\cdot 931{,}5\text{MeV}/c^2}} = \underline{0{,}1\text{eV}}$

$E_{n=0} = 5{,}2\text{eV} - 0{,}1\text{eV} = \underline{5{,}1\text{eV}}$ grunntilstanden

4.3



a) Skulle vise at vi får dette når vi har $m_1 r_1 = m_2 r_2$

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r)$$

$|p_1| = |p_2|$

Bevis: $\mathcal{H} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(r) = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} + V(r)$

$= \frac{p^2}{2}\frac{m_1+m_2}{m_1 m_2} = \frac{p^2}{2\mu}$ QED

b) $\hat{\mathcal{H}} = \frac{-\hbar^2}{2\mu}\cdot\nabla^2 + V(r)$

$= \frac{-\hbar^2}{2\mu}\left(\underbrace{\frac{1}{r_0^2}\frac{\partial}{\partial r}r_0^2\frac{\partial}{\partial r}}_{\text{konstant}} - \frac{1}{\hbar^2 r_0^2}\hat{L}^2\right) + \underbrace{V(r_0)}_{\text{konstant}}$

$= \frac{-\hbar^2}{2\mu}\left(-\frac{\hat{L}^2}{\hbar^2 r_0^2}\right) + \text{konstantene}$

$= \underline{\frac{\hat{L}^2}{2\mu r_0^2}}$

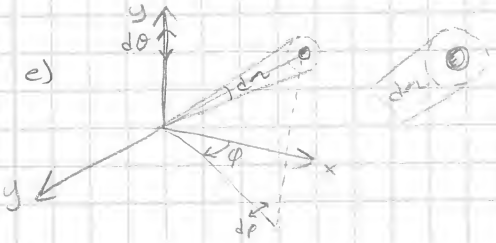
c) Det er rotasjons bevegelse den utfører.

d) $\hat{L}^2\psi_E = \hbar^2 l(l+1)\psi_E$ $\quad H\psi_E = \frac{\hat{L}^2}{2\mu r_0^2}\psi_E = \underline{\frac{\hbar^2 l(l+1)}{2\mu r_0^2}\psi_E}$

kap. 8.3. $[\hat{L}_z, \hat{L}^2] = 0$ og $[\hat{L}_z, \hat{H}] = 0$

L_z kan også ha skarpe verdier. Ser bort fra konstanten.

e)



$|\psi_E|^2 \underbrace{\sin\theta\, d\theta\, d\varphi}_{} =$ angular probability

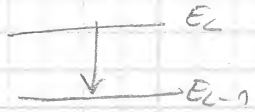
$|\psi_E|^2 dr$ (dr romvinkel)

$\psi_E = \Theta(\theta)\Phi(\varphi)$

f) $\hat{\mathcal{H}} = \frac{\hat{L}^2}{2\mu r_0^2} - D\hat{L}^4 \iff \mathcal{H}\psi = \frac{\hat{L}^2}{2\mu r_0^2}\psi - D\hat{L}^4\psi$

$$= \underline{\underline{\frac{l(l+1)\hbar^2\psi}{2\mu r_0} - D(l(l+1)\hbar^2)^2\psi}}$$

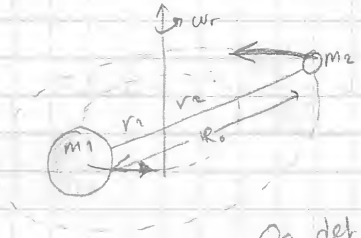
g) $\Delta L = \pm 1$



$$E_L - E_{L-1} = \frac{\hbar^2}{2\mu r_0^2}\left(l(l+1) - (l-1)l\right) - D\hbar^4\left((l(l+1))^2 - ((l-1)l)^2\right)$$

$$= \underline{\underline{\frac{-\hbar^2}{2\mu r_0^2}(2l) - D\hbar^4(4l^3) = h\nu}}$$

4.4



a) Vi har at $I_{cm} = m_1 r_1^2 + m_2 r_2^2$

Vi får at: $\left.\begin{array}{l} r_1 + r_2 = R_0 \\ m_1 r_1 - m_2 r_2 = 0 \end{array}\right\} \rightarrow \left\{\begin{array}{l} r_1 = \frac{m_2}{m_1+m_2} R_0 \\ r_2 = \frac{m_1}{m_1+m_2} R_0 \end{array}\right.$

Og det gir da:

$$\underline{I_{cm}} = m_1\left(\frac{m_2}{m_1+m_2}R_0\right)^2 + m_2\left(\frac{m_1}{m_1+m_2}R_0\right)^2 = \frac{m_1 m_2}{m_1+m_2}R_0^2 = \underline{\mu R_0^2}$$

hvor μ er systemets reduserte masse. Systemets banespinn er gitt ved

(B) $(v = \omega R)$ $\quad L = r_1(m_1 v_1) + r_2(m_2 v_2) = m_1 \omega r_1^2 + m_2 \omega r_2^2 = \omega(I_1 + I_2) = \underline{I_{cm}\omega}$

Systemet totale energi er

$$E_{rot} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\frac{I_{cm}^2\omega^2}{I_{cm}} = \underline{\underline{\frac{L_{cm}^2}{2I_{cm}}}}$$

b) Hamilton operatoren! $\quad \hat{\mathcal{H}}\psi = E\psi$

Vi setter inn den kvantemekaniske operatoren for L i det klassiske uttrykk for E_{rot} og får

$$[E_{rot}] = \frac{1}{2I_{cm}}[L^2]$$

$$\frac{\hat{L}_{cm}^2}{2I_{cm}}\psi = E_{rot}\psi$$

$$\frac{1}{2I_{cm}}l(l+1)\hbar^2\psi = E_{rot}\psi \quad \leftarrow \text{egenverdi ligningen}$$

med egenverdiene: $\quad E_{rot}(l) = \frac{1}{2I_{cm}}l(l+1)\hbar^2$

$$\Delta E_{rot} = E_{rot}(l+1) - E_{rot}(l)$$

$$= \frac{(l+1)(l+1+1)\hbar^2}{2I} - \frac{l(l+1)\hbar^2}{2I}$$

$$= \frac{l^2+3l+2-l^2-l}{2I}\hbar^2 = \frac{l+1}{I_{cm}}\hbar^2$$

$$\hbar^2\frac{l(l+1)}{2I} - \frac{(l-1)(l-1+1)}{2I}\hbar^2$$

$$= \frac{l^2+l-l^2+l}{2I}\hbar^2$$

$$= \frac{l\hbar^2}{2I_{cm}}$$

c) Co-molekyl

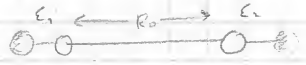
grunntilstanden $\rightarrow l=0$ eksitert tilstand $l=1$

$$E_\gamma = h\nu = E_{rot}(1) - E_{rot}(0)$$

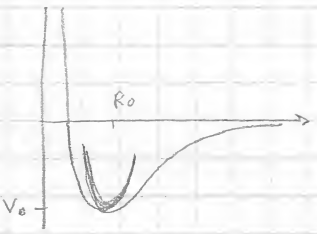
$$h\nu = \frac{2}{I_{cm}}\hbar^2 - \frac{1}{I_{cm}}\hbar^2 = \frac{1}{I_{cm}}\hbar^2 \Leftrightarrow I_{cm} = \frac{\hbar^2}{h\nu} = \frac{(6{,}58\cdot10^{-16}\,eV\cdot s)^2}{4{,}76\cdot10^{-4}\,eV} = 9{,}108\cdot10^{-28}\,eV\cdot s^2$$

$$= 1{,}450\cdot10^{-46}\,kgm^2$$

d)



$$\xi = R - R_0 = \xi_1 + \xi_2$$



$$V(R) = V(R_0) + (R-R_0)\left.\frac{dV}{dR_0}\right|_{R=R_0} + \frac{1}{2}\left.\frac{d^2V}{dR^2}\right|_{R=R_0}(R-R_0)^2$$

$$V(\xi) = \frac{1}{2}\left.\frac{d^2V}{dR^2}\right|_{R=R_0} = \frac{1}{2}k^2\xi \qquad \xi = R - R_0$$

altså:

$$\frac{-\hbar^2}{2\mu}\frac{d^2}{d\xi^2}\psi(\xi) + \frac{1}{2}k\xi^2\psi(\xi) = E_{vib}\psi(\xi)$$

e) Energi egenverdiene:

$$E_{vib}(n) = (n+1/2)\hbar\omega_0 \text{ og hvor } \omega_0 = \sqrt{\frac{k}{\mu}}$$

Energi avstanden mellom to nivåer blir

$$E_{vib}(n) - E_{vib}(n-1) = [(n+1/2) - (n-1+1/2)]\hbar\omega_0$$

$$= [n+1/2 - n + 1/2]\hbar\omega_0$$

$$= \hbar\omega_0$$

Fra Heisenbergs usikkerhetsrelasjonen $\Delta x\Delta p \geq \frac{\hbar}{2}$, De to atomene har $\Delta x \approx R_0$. Dette gir $\Delta p \geq \frac{\hbar}{2R_0}$. Da må bevegelsesmengde $p>0$ og systemet får en nullpunkts energi $E_{vib}(\text{grunntilstand}) = p^2/2\mu + E_p > 0$

f) Molekylets totale rotasjons pluss vibrasjons energi blir

$$E = E_{rot} + E_{vib} = \frac{\hbar^2}{2I_{cm}} l(l+1) + n(n+1/2)\hbar\omega_p$$

g) Utvalgsreglene er: $\Delta n = \pm 1$
$\Delta L = \pm 1$
$\Delta m_l = 0, \pm 1$

h)

$(n=0, l=0) \rightarrow (n=1, l=1)$: $\Delta E = \hbar\omega_p + \hbar^2/2I_{cm} = 0{,}2657\,eV$

$(n=0, l=1) \rightarrow (n=1, l=0)$: $\Delta E = \hbar\omega_p - \hbar^2/2I_{cm} = 0{,}2648\,eV$

$\rightarrow (n=1, l=2)$: $\Delta E = \hbar\omega_p + 2\hbar^2/2I_{cm} = 0{,}2662\,eV$

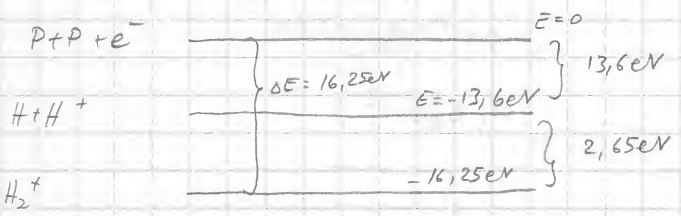
$(n=0, l=2) \rightarrow (n=1, l=1)$: $\Delta E = \hbar\omega_p - 2\hbar^2/2I_{cm} = 0{,}2643\,eV$

$(n=1, l=3)$: $\Delta E = \hbar\omega_p + 3\hbar^2/2I_{cm} = 0{,}2669\,eV$

4.1

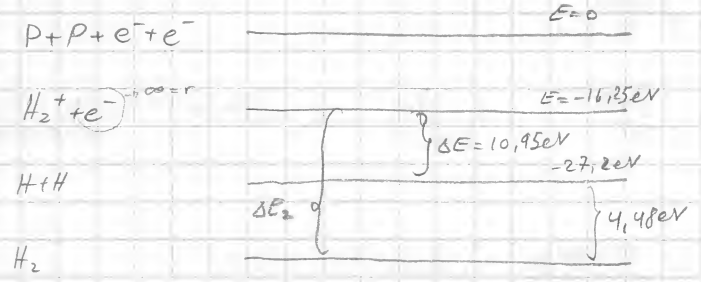
$$E(H_2^+) = E(H + H^+) - 2{,}65\,eV$$

a)



$$\Delta E = 13{,}6\,eV + 2{,}65\,eV = \underline{16{,}25\,eV}$$

b)



$E(H+H) = -13{,}6\,eV \cdot 2 = -27{,}2\,eV$

$\Delta E = E(H_2^+ + e^-) - E(H+H)$

$= -16{,}25\,eV - (-27{,}2\,eV)$

$= \underline{10{,}95\,eV}$

c) $E(H_2) = E(H+H) - 4{,}48\,eV$

$\Delta E_2 = E(H_2 + e^-) - E(H_2) = (10{,}95 + 4{,}48)\,eV = \underline{15{,}43\,eV}$

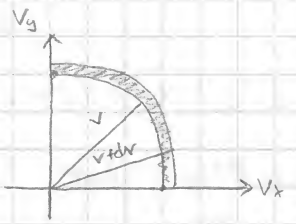
4.5

a) $\rho = 1{,}7 \cdot 10^{-8}\,\Omega m$ egenvekten : $8{,}96\,g/cm^3$ $\sigma = \frac{1}{\rho}$ Sigma

$\frac{\text{antall elektroner}}{\text{vol-enhet}}$ $n = \frac{1\,\text{elektron}}{\text{atom}} \cdot \frac{6{,}02 \cdot 10^{23}\,\text{atomer}}{\text{mol}} \cdot \frac{1\,\text{mol}}{63{,}5\,g} \cdot \frac{8{,}96\,g}{cm^3}$

$= \underline{8{,}45 \cdot 10^{22}\,\text{elektroner}/cm^3}$

b) $\rho = \frac{1}{\sigma} = \frac{mc\,\bar{v}}{ne^2L}$ klassiske modellen



$F_{MB} = Ae^{-\frac{E}{kT}}$

$n = N/V = \int g(E)\,F_{MB}(E)\,dE$

hvor $g(E)\,dE = 4\pi v^2 dv$

$\Rightarrow = \int_0^\infty 4\pi v^2 e^{-\frac{mv^2}{2kT}}\,dv$, $\frac{mv^2}{2kT} = t$ $dt = \frac{m}{kT} v\,dv$

$$\frac{N}{V} = A\cdot 2\pi\left(\frac{2kT}{m}\right)^{3/2}\int_0^\infty t^{1/2}e^{-t}\,dt = A\cdot 2\pi\left(\frac{2kT}{m}\right)^{3/2}\Gamma(3/2)$$

$$= A\cdot 2\pi\left(\frac{2kT}{m}\right)^{3/2}\cdot\frac{1}{2}\sqrt{\pi}$$

$$= A\pi^{3/2}\left(\frac{2kT}{m}\right)^{3/2}$$

$$= A\left(\frac{2\pi kT}{m}\right)^{3/2}$$

$$n(v)dv = g(E)F_{MB}(E)dv = 4\pi\frac{N}{V}\left(\frac{m}{2\pi kT}\right)^{3/2}v^2 e^{-\frac{mv^2}{2kT}}dv$$

$$\bar{v} = \int v\,n(v)\,dv \qquad v_{rms} = \sqrt{\frac{2kT}{m}}$$

$$\overline{v^2} = \int v^2 n(v)\,dv$$

$$v_{rms} = \sqrt{\overline{v^2}} \qquad v_{rms} = \sqrt{\frac{3kT}{m_e}} = c\sqrt{\frac{3kT}{m_e c^2}} = c\cdot 0{,}39\cdot 10^{-3} = \underline{1{,}2\cdot 10^5\ m/s}$$

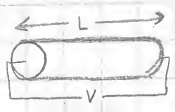
$$L = \frac{m_e\bar{v}}{ne^2}\frac{1}{S} = 3{,}9\cdot 10^{-8}\,m = \underline{15000\,a}\ , \text{ hvor } a = 2{,}6\cdot 10^{-10}\,m$$

c) $$A = \frac{1}{e^{(E-E_F)/kT}-1} \qquad E_F = 7eV = \frac{1}{2}mv_F^2$$

$\underline{v_F = 1{,}5\cdot 10^6\ m/s}$ de deltar i strømmen

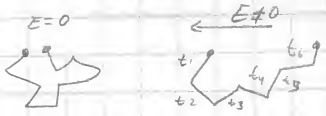
hvis vi skal beregne L, så bruk $\bar{v}_F$: $L = \frac{m\bar{v}_F}{ne^2}\frac{1}{S} = L_M\cdot\frac{\bar{v}_E}{v_{rms}} = \underline{12{,}5\,L_M}$

$$\sigma = \frac{ne^2L}{m_e\bar{v}}\ , \quad V = \frac{I}{R} \Leftrightarrow I = V/R$$



$$\underline{J = \frac{I}{A} = \frac{V}{RA} = \frac{l}{RA}\frac{V}{l} = \sigma E}$$

$J = n_e v_d$



$L = \vec{v}\cdot\tau$ hvor τ er tiden mellom hver kollisjonen.

$S = v_d\cdot\tau\cdot p$ hvor p er antall kollisjoner.

$$S = \frac{1}{2}a\left(t_1^2+t_2^2+t_3^2+t_4^2+\cdots+t_p^2\right), \text{ hvor } a = \frac{eE}{m_e}$$

$$= \frac{1}{2}a\,\overline{t^2}\cdot p \text{ , hvor } \overline{t^2} = \frac{1}{p}\left(t_1^2+t_2^2+t_3^2+t_4^2+\cdots t_p^2\right)$$

$$n(t)dt = \frac{Ne^{-t/\tau}}{\tau}dt \quad \text{antall kollisjoner}$$

$$\int_0^\infty n(t)\,dt = N \qquad \bar{t} = \frac{1}{N}\int_0^\infty t\,n(t)\,dt = \tau$$

$$\overline{t^2} = \frac{1}{N}\int_0^\infty A^2 n(t)\,dt = \underline{2\tau^2}$$

$$S = \frac{1}{2} a p (2\tau^2) = \frac{nE}{m_e} p\tau^2 = pv_d \tau \quad \text{, hvor } v_d = \frac{eE\tau}{m_e}$$

$$J = \frac{ne^2\tau}{m_e} E = \sigma E \quad , \sigma = \frac{ne^2\tau}{m_e} \qquad \tau \cdot \bar{v} = l$$

$$= \frac{ne^2 l}{m_e v}$$

2.14

a) Tidsavhengige Schrödinger ligning

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi(x,t) + V(x,t)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

b) Setter $\Psi(x,t) = \Psi(x)\varphi(t)$

Bruker tidsavhengige Schrödinger ligning og deler med $\Psi(x)\varphi(t)$

Potensialet $V(x,t)$ er ikke tidsavhengig. Derfor kan vi sette at potensialet er lik $V(x)$.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi(x)\varphi(t) + V(x)\Psi(x)\varphi(t) = i\hbar\frac{d}{dt}\Psi(x)\varphi(t)$$

$$-\frac{\hbar^2}{2m}\varphi(t)\frac{d^2}{dx^2}\Psi(x) + V(x)\Psi(x)\varphi(t) = i\hbar\Psi(x)\frac{d}{dt}\varphi(t) \quad \Big| \cdot \frac{1}{\Psi(x)\varphi(t)}$$

$$-\frac{\hbar^2}{2m}\frac{1}{\Psi(x)}\frac{d^2}{dx^2}\Psi(x) + V(x) = i\hbar\frac{1}{\varphi(t)}\frac{d}{dt}\varphi(t)$$

$$-\frac{\hbar^2}{2m}\frac{1}{\Psi(x)}\frac{d^2}{dx^2}\Psi(x) + V(x) = E \quad \Big| \cdot \Psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi(x) + V(x)\Psi(x) = E\Psi(x)$$

Finne $\varphi(t)$.

$$\hat{H}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) \quad \Leftrightarrow \quad \hat{H}\Psi(x)\varphi(t) = i\hbar\frac{\partial}{\partial t}\Psi(x)\varphi(t)$$

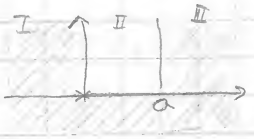
$$\hat{H}\varphi(t) = i\hbar\frac{\partial}{\partial t}\varphi(t) \quad \Leftrightarrow \quad \frac{E}{i\hbar}\varphi(t) = \frac{d}{dt}\varphi(t)$$

$$\frac{E}{i\hbar}dt = \frac{1}{\varphi(t)}d\varphi(t) \quad \Leftrightarrow \quad \int\frac{E}{i\hbar}dt = \int\frac{1}{\varphi(t)}d(\varphi(t))$$

$$\frac{Et}{i\hbar} = \ln|\varphi(t)| \quad \Leftrightarrow \quad e^{\frac{Et}{i\hbar}} = \varphi(t)$$

$$\varphi(t) = e^{-\frac{iEt}{\hbar}}$$

c) Potensielle energien: $V(x) = \begin{cases} +\infty & x<0 \\ 0 & 0 \le x \le a \\ +\infty & x>a \end{cases}$



Finne egenfunksjonene:

I områdene der potensialet er ∞ er Ψ_I og Ψ_{III} lik 0, altså $\Psi_I = \Psi_{III} = 0$

I område II gir oss schrödingers ligningen:

$$\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = E\psi(x)$$

$$\frac{d^2}{dx^2}\psi(x) = -\frac{2mE}{\hbar^2}\psi(x) = -k^2\psi(x) \quad , \text{hvor } k^2 = \frac{2mE}{\hbar^2}$$

$$\Psi_{II}(x) = Ae^{ikx} + Be^{-ikx}$$

Må bestemme konstantene, ved å benytte kravene:

1) $\Psi_I(0) = \Psi_{II}(0)$ og 2) $\Psi_I'(0) = \Psi_{II}'(0)$

Slik at det gir oss 1) $0 = Ae^{ikx} + Be^{-ikx}$

$Ae^{ikx} = -Be^{ikx}$ $(x=0) \Rightarrow Ae^0 = -Be^0 \Leftrightarrow \underline{A = -B}$

altså:

$$\Psi_{II}(x) = A(e^{ikx} - e^{ikx})$$

$$= A(\cos kx + i\sin kx - \cos kx + i\sin kx)$$

$$= 2Ai\sin kx = \underline{C\sin kx}$$

2) $0' = (Ae^{ikx} + Be^{-ikx})' \Leftrightarrow 0 = ikA - ikB \Leftrightarrow \underline{A = B}$ $(x=0)$

altså:

$$\Psi_{II}(x) = A(e^{ikx} + e^{-ikx})$$

$$= A(\cos kx + i\sin kx + \cos kx - i\sin kx)$$

$$= 2A\cos kx = \underline{C\cos kx}$$

Tilsvarende for kravene:

1) $\Psi_{II}(a) = \Psi_{III}(a)$ og 2) $\Psi_{II}'(a) = \Psi_{III}'(a)$

1) $C \sin kx = 0 \Leftrightarrow \sin ka = \frac{0}{C} \Leftrightarrow \sin ka = 0$

Da må: $ka = n\pi$, hvor $n = 0, 1, 2, 3$ osv

for å få alltid null på høyreside.

altså kvantiseringsbetingelse $\underline{ka = n\pi}$

tilsvarende for: $C\cos kx = 0 \Leftrightarrow \cos ka = 0$, $ka = (\frac{1}{2} + n)\pi$

hvor $n = 0, 1, 2, 3$ osv.

Energi egenverdiene: Det får vi ved følgende ligningen. Vi hadde

$$k^2 = \frac{2mE}{\hbar^2} \quad \text{og} \quad ka = n\pi$$

$$\Downarrow$$

$$(ka)^2 = (n\pi)^2 \Leftrightarrow k^2 = \frac{n^2\pi^2}{a^2}$$

altså:

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \underline{\frac{\hbar^2\pi^2}{2ma^2} n^2}$$

Finne de normerte energi egenfunksjonene $\Psi(x)$.

$$1 = \int_0^a \Psi^*(x)\,\Psi(x)\,dx = \int_0^a C^2 \sin^2 kx\,dx = \int_0^a C^2\left(\frac{1}{2} - \frac{1}{2}\cos 2kx\right) dx$$

$$= C^2\left[\frac{1}{2}x - \frac{1}{4k}\sin 2kx\right]_0^a = C^2\left[\frac{1}{2}a - \frac{1}{4k}\sin 2ka\right] \quad ka = n\pi$$

$$= C^2\left[\frac{1}{2}a - \frac{1}{4k}\underbrace{\sin 2n\pi}_{0}\right] = \underline{\frac{C^2}{2}a} \Leftrightarrow \frac{C^2}{2}a = 1 \Leftrightarrow \underline{C = \sqrt{\frac{2}{a}}}$$

Normerte egenfunksjonene:

$$\Psi_I(x) = \Psi_{III}(x) = 0 \quad \text{og}$$

$$\underline{\Psi_{II}(x) = \sqrt{2/a}\,\sin\left(\frac{n\pi x}{a}\right)}$$

d) Finne forventningsverdien x, altså $\langle x \rangle$

$$\Rightarrow \langle x \rangle = \int_0^a \psi^* x \psi \, dx = \int_0^a \left(\sqrt{\frac{2}{a}}\right)^2 \sin^2\left(\frac{n\pi x}{a}\right) x \, dx = \frac{2}{a}\int_0^a x \sin^2(kx)\, dx$$

$$= \frac{2}{a}\int_0^a \left(\frac{1}{2}x - \frac{1}{2}x\cos 2kx\right) dx = \frac{1}{a}\int_0^a (x - x\cos 2kx)\, dx \qquad \text{(se formel i teksten)}$$

$$= \frac{1}{a}\left[\frac{1}{2}x^2 - \left(\frac{1}{(2k)^2}\cos 2kx + \frac{x}{2k}\sin 2kx\right)\right]_0^a =$$

$$= \frac{1}{a}\left[\frac{1}{2}a^2 - \frac{1}{4k^2}\cos 2ka - \frac{a}{2k}\sin 2ka + \frac{1}{(2k)^2}\cos 2k\cdot 0 + 0\right]$$

$$= \frac{1}{a}\left[\frac{1}{2}a^2 - \frac{1}{4k^2}\cos\left(2\frac{n\pi a}{a}\right) - 0 + \frac{1}{4k^2}\cos\left(2\frac{n\pi \cdot 0}{a}\right)\right]$$

$$= \frac{1}{a}\left[\frac{1}{2}a^2 - \frac{1}{4k^2}\cos(2n\pi) + \frac{1}{4k^2}\right] = \frac{1}{a}\left[\frac{1}{2}a^2 - 1 + 1\right] = \underline{\underline{\frac{1}{2}a}}$$

$$\Rightarrow \langle x^2 \rangle = \frac{2}{a}\int_0^a x^2 \sin^2(kx)\, dx = \frac{2}{a}\int_0^a x^2\left(\frac{1}{2} - \frac{1}{2}\cos 2kx\right) dx = \frac{2}{a}\int_0^a \left(\frac{1}{2}x^2 - \frac{1}{2}x^2\cos 2kx\right) dx$$

$$= \frac{1}{a}\left[\frac{x^3}{3}\right]_0^a - \frac{1}{a}\int_0^a x^2\cos 2kx\, dx = \text{Bruker:}$$

$$\int x^2\cos ax\, dx = \frac{2x}{a^2}\cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right)\sin ax + C$$

$$= \frac{1}{3}a^2 - \frac{1}{a}\left[\frac{2x}{4k^2}\cos 2kx + \left(\frac{x^2}{2k} - \frac{2}{8k^3}\right)\sin 2kx\right]_0^a$$

$$= \frac{1}{3}a^2 - \frac{1}{a}\left(\frac{2a}{4k^2}\cos 2ka + 0 - (0+0)\right) = \frac{1}{3}a^2 - \frac{1}{a}\frac{2a}{4k^2}1 = \underline{\underline{\frac{1}{3}a^2 - \frac{1}{2k^2}}}$$

$$\Rightarrow \langle \hat{p} \rangle = \int_0^a \psi^*(x)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi(x)\, dx = \frac{2}{a}\int_0^a \sin kx\,(-i\hbar)\,k\cos kx\, dx$$

$$= \frac{-i\hbar}{a}\int_0^a 2\sin kx\cos kx\, dx = \frac{-i\hbar}{a}\int_0^a \sin 2kx\, dx = \frac{-i\hbar}{a}\left[-\frac{1}{2k}\cos 2kx\right]_0^a$$

$$= \frac{-i\hbar}{a}\left[-\frac{1}{2k}\cos 2ka + \frac{1}{2k}\cos 2k\cdot 0\right] = \frac{-i\hbar}{a}\left[-1+1\right] = \underline{\underline{0}}$$

$$\langle \hat{p} \rangle^2 = \int_0^a \psi^* (-i\hbar \frac{\partial}{\partial x})^2 \psi \, dx = \int_0^a \left(\sqrt{\frac{2}{a}}\right)^2 \sin kx \, \left((-i)^2 \hbar^2 \frac{\partial^2}{\partial x^2} \sin kx\right) dx$$

$$= \frac{2}{a} \int_0^a \sin kx \, (-i\hbar)^2 (-k^2 \sin kx) \, dx = \frac{2\hbar^2 k^2}{a} \int_0^a \sin^2 kx \, dx$$

$$= \frac{2\hbar^2 k^2}{a} \int_0^a \left(\frac{1}{2} - \frac{\cos 2kx}{2}\right) dx = \frac{\hbar^2 k^2}{a} \left[x - \frac{1}{2k} \sin 2kx\right]_0^a = \underline{\hbar^2 k^2}$$

e) Heisenbergs usharphets relasjon $\Delta x \, \Delta p \geq \frac{\hbar}{2}$

Def: $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ og $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$

$$\Delta x = \sqrt{\frac{1}{3}a^2 - \frac{1}{2n^2\pi^2}a^2 - \frac{1}{4}a^2} = \underline{\sqrt{\frac{1}{12}a^2 - \frac{a^2}{2n^2\pi^2}}}$$

$$\Delta p = \sqrt{\hbar^2 k^2} = \hbar k = \underline{\frac{\hbar n \pi}{a}}$$

Slik at det blir:

$$\Delta x \Delta p = \sqrt{\frac{1}{12}a^2 - \frac{a^2}{2n^2\pi^2}} \cdot \frac{\hbar n \pi}{a} = a\sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}} \frac{\hbar n \pi}{a}$$

$$= \hbar n \pi \sqrt{\frac{1}{12} - \frac{1}{2n^2\pi^2}}$$

For grunntilstand $n = 1$

$$= \hbar \pi \sqrt{\frac{1}{12} - \frac{1}{2\pi^2}} = \hbar \sqrt{\frac{\pi^2}{12} - \frac{1}{2}} \approx 0{,}567\,\hbar > \frac{\hbar}{2} \quad \text{Q.E.D}$$

f) $\Psi(x,0) = \frac{1}{\sqrt{2}} (\psi_1(x) - \psi_2(x))$

$$\Psi(x,t) = \frac{1}{\sqrt{2}} \left(\psi_1(x) e^{\frac{-iE_1 t}{\hbar}} - \psi_2(x) e^{\frac{-iE_2 t}{\hbar}}\right)$$

$\psi_1(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$ og $\psi_2(x) = \sqrt{\frac{2}{a}} \sin \frac{2\pi x}{a}$

$$\Psi(x,t) = \frac{1}{\sqrt{a}} \left(\sin \frac{\pi x}{a} e^{\frac{-iE_1 t}{\hbar}} - \sin \frac{2\pi x}{a} e^{\frac{-iE_2 t}{\hbar}}\right)$$

$E_1 = \frac{\hbar^2 \pi^2}{2ma^2}$ $\quad E_2 = \frac{\hbar^2 \pi^2}{2ma^2} 4 \Rightarrow E_2 = 4E_1$

$$\Psi(x,t) = \frac{1}{\sqrt{a}} \left(\sin \frac{\pi x}{a} e^{\frac{-iE_1 t}{\hbar}} - \sin \frac{2\pi x}{a} e^{\frac{-4iE_1 t}{\hbar}}\right)$$

Forventningsverdien av $\langle x \rangle$.

$$\langle x \rangle = \int_0^a \Psi(x,t)^* \, x \, \Psi(x,t)\, dx = \frac{1}{a}\int_0^a x\left[\left(\sin\frac{\pi x}{a} e^{\frac{iE_1 t}{\hbar}} - \sin\frac{2\pi x}{a} e^{\frac{4iE_1 t}{\hbar}}\right)\left(\sin\frac{\pi x}{a} e^{-\frac{iE_1 t}{\hbar}} - \sin\frac{2\pi x}{a} e^{-\frac{4iE_1 t}{\hbar}}\right)\right] dx$$

$$= \frac{1}{a}\int_0^a x\left(\sin^2 kx + \sin^2 2kx - \sin kx \cdot \sin 2kx\, e^{-\frac{3iE_1 t}{\hbar}} - \sin 2kx \sin kx\, e^{+\frac{3iE_1 t}{\hbar}}\right) dx$$

$$= \frac{1}{a}\int_0^a x\left(\sin^2 kx + \sin^2 2kx - \sin kx \sin 2kx\left(e^{\frac{3iE_1 t}{\hbar}} + e^{-\frac{3iE_1 t}{\hbar}}\right)\right) dx$$

mellomregning av siste leddet:

$$e^{\frac{3iE_1 t}{\hbar}} + e^{-\frac{3iE_1 t}{\hbar}} = \cos\left(\frac{3E_1 t}{\hbar}\right) + i\sin\left(\frac{3E_1 t}{\hbar}\right) + \cos\left(\frac{3E_1 t}{\hbar}\right) - i\sin\left(\frac{3E_1 t}{\hbar}\right) = 2\cos\left(\frac{3E_1 t}{\hbar}\right)$$

$$= \frac{1}{a}\int_0^a x\left(\sin^2 kx + \sin^2 2kx - \sin kx \sin 2kx\left(2\cos\left(\frac{3E_1 t}{\hbar}\right)\right)\right) dx$$

$$= \frac{1}{a}\int_0^a x\left(\frac{1}{2} - \frac{\cos 2kx}{2} + \frac{1}{2} - \frac{\cos 4kx}{2}\right) dx - \frac{1}{a}\int_0^a x \sin kx \sin 2kx\, 2\cos\left(\frac{3E_1 t}{\hbar}\right) dx$$

$$= \frac{1}{2a}\int_0^a \left(x - x\cos 2kx + x - x\cos 4kx\right) dx - \frac{2}{a}\int_0^a x \sin kx \sin 2kx \cos\left(\frac{3E_1 t}{\hbar}\right) dx$$

$$= \frac{1}{2a}\int_0^a \left(2x - x\cos 2kx - x\cos 4kx\right) dx - \frac{2}{a}\int_0^a x \sin kx \sin 2kx \cos\left(\frac{3E_1 t}{\hbar}\right) dx$$

$$= \frac{1}{2a}\left[x^2 - \frac{1}{4k^2}\cos 2kx - \frac{x}{2k}\sin 2kx - \frac{1}{16k^2}\cos 2kx - \frac{x}{4k}\sin 4kx\right]_0^a - \cdots$$

$$= \frac{1}{2a}\left[a^2 - \frac{1}{4k^2}\cos 2\frac{\pi}{a}a - \frac{1}{16k^2}\cos 2\frac{\pi}{a}a + \left(0 - \frac{1}{4k^2}\cos 0 - \frac{1}{16}\cos 2\pi\right)\right] - \cdots$$

$$= \frac{a}{2} - \frac{2}{a}\int_0^a x \sin kx \sin 2kx \cos\left(\frac{3E_1 t}{\hbar}\right) dx = \frac{a}{2} - \cos\left(\frac{3E_1 t}{\hbar}\right)\int_0^a \frac{2x}{a}\sin\frac{\pi x}{a}\sin\frac{2\pi x}{a}\, dx$$

$$= \frac{a}{2} - \frac{1}{a}\cos\left(\frac{3E_1 t}{\hbar}\right)\int_0^a x\left(\cos\left(\frac{-\pi x}{a}\right) - \cos\left(\frac{3\pi x}{a}\right)\right) dx = \frac{a}{2} - \frac{1}{a}\cos\frac{3E_1 t}{\hbar}\int_0^a x\cos\frac{\pi x}{a} - x\cos\frac{3\pi x}{a}\, dx$$

$$= \frac{a}{2} - \frac{1}{a}\cos\frac{3E_1 t}{\hbar}\left[\frac{a^2}{\pi^2}\cos\frac{\pi}{a}x + \frac{xa}{\pi}\sin\frac{\pi x}{a} - \frac{a^2}{9\pi^2}\cos\frac{3\pi}{a} - \frac{xa}{3\pi}\sin\frac{3\pi}{a}x\right]_0^a =$$

$$= \frac{a}{2} - \frac{1}{a}\cos\frac{3E_1 t}{\hbar}\left[-\frac{a^2}{\pi^2} - \frac{a^2}{9\pi^2} - \frac{a^2}{\pi^2} + \frac{a^2}{9\pi^2}\right] = \frac{a}{2} - \cos\frac{3E_1 t}{\hbar}\left[-\frac{2a}{\pi^2} + \frac{2a}{9\pi^2}\right]$$

$$= \frac{a}{2} - \cos\frac{3E_1 t}{\hbar}\left[-\frac{18a}{9\pi^2} + \frac{2a}{9\pi^2}\right] = \frac{a}{2} + \frac{16}{9\pi^2}\cos\frac{3E_1 t}{\hbar}$$

3.3

$$R_{20}(r) = N_{20}\left(2 - \frac{r}{a_0}\right)\exp\left(-r/2a_0\right)$$

$$R_{21}(r) = N_{21}\,\frac{r}{a_0}\exp\left(-r/2a_0\right)$$

a) Bestemme konstantene N_{20} og N_{21}.

$$1 = \int_0^\infty |R(r)|^2 r^2 dr = N_{20}^2 \int_0^\infty (2 - r/a_0)^2 \exp(-r/a_0) r^2 dr$$

$$= N_{20}^2 \int_0^\infty \left(4 - 4r/a_0 + r^2/a_0^2\right) r^2 \exp(-r/a_0)\, dr = N_{20}^2 \int_0^\infty \left(4r^2 - \frac{4r^3}{a_0} + \frac{r^4}{a_0^2}\right)\exp(-r/a_0) dr$$

$\rho = \frac{r}{a_0}$, $d\rho = \frac{1}{a_0} dr$

$$= N_{20}^2 \int_0^\infty \left(4a_0^2\rho^2 - 4\rho^3 a_0^2 + \rho^4 a_0^2\right)\exp(-\rho)\, a_0 d\rho$$

$$= N_{20}^2 a_0^3 \int_0^\infty \left(4\rho^2 - 4\rho^3 + \rho^4\right)\exp(-\rho) d\rho = N_{20}^2 a_0^3 \int_0^\infty \left(4\rho^2 - 4\rho^3 + \rho^4\right) e^{-\rho} d\rho$$

$$= N_{20}^2 a_0^3 \left[4\Gamma(3) - 4\Gamma(4) + \Gamma(5)\right]$$

$$= N_{20}^2 a_0^3 \left[4 \cdot 2! - 4 \cdot 3! + 4!\right]$$

$$1 = N_{20}^2 a_0^3 \left[8 - 24 + 24\right] = 8 N_{20}^2 a_0^3$$

$$\frac{1}{8a_0^3} = N_{20}^2 \iff \underline{\underline{N_{20} = \frac{1}{2a_0\sqrt{2a_0}}}}$$

Tilsvarende for N_{21}:

$$1 = \int_0^\infty |R(r)|^2 r^2 dr = N_{21}^2 \int_0^\infty \left(\frac{r}{a_0^2}\right)^2 r^2 \exp(-r/a_0) dr$$

$$= N_{21}^2 \int_0^\infty \rho^2 \rho^2 a_0^2 \exp(-\rho) a_0 d\rho = N_{21}^2 a_0^3 \int_0^\infty \rho^4 \exp(-\rho) d\rho$$

$$= N_{21}^2 a_0^3 \Gamma(5) = N_{21}^2 a_0^3 4! = 24 N_{21}^2 a_0^3$$

$$1 = 24 a_0^3 N_{21}^2 \iff \underline{\underline{N_{21} = \frac{1}{\sqrt{24 a_0^3}}}}$$

b) Skal finne når sannsynligheten er størst for å finne elektronet når hydrogenet antas å være i p-tilstanden?

$$R_{21}(r) = N_{21}\frac{r}{a_0}\exp(-r/a_0)$$

Regner ut $\frac{dP}{dr} = 0$ der $P = |R_{21}(r)|^2 r^2$

$$|R_{21}(r)|^2 r^2 = N_{21}^2 \frac{r^4}{a_0^2}\exp(-r/a_0)$$

$$\frac{dP}{dr} = \frac{N_{21}^2}{a_0^2}\left(4r^3\exp(-r/a_0) + r^4\left(-\frac{1}{a_0}\right)\exp(-r/a_0)\right) = 0$$

$$= \frac{N_{21}^2}{a_0^2}\exp(-r/a_0)\left(4r^3 - r^4/a_0\right) = 0$$

$$r^3(4 - r/a_0) = 0$$

$$\underline{r = 0} \quad V \quad \underline{r = 4a_0}$$

Bohrs atommodell: $r_n = n^2 a_0$

Her er $n = 2 \rightarrow \underline{r = 4a_0}$ stemmer overens med det vi har regnet ut.

Sannsynligheten er gitt ved

$$P = \int_0^{a_0} |R(r)|^2 r^2 dr = N_{21}^2 \int_0^{a_0} \frac{r^2}{a_0^2}\exp(-r/a_0)\, r^2 dr$$

$\rho = \frac{r}{a_0}$

$\rho a_0 = r$

$\Downarrow$

$r = a_0 \Rightarrow \rho = 1$

$r = 0 \Rightarrow 0$

VIKTIG!

$$= \int_0^{a_0} N_{21}^2 \frac{r^4}{a_0^2}\exp(-r/a_0)dr = \int_0^{1} N_{21}^2 \rho^2 \rho^2 a_0^2 \exp(-\rho)\, a_0 d\rho$$

$$= \int_0^1 N_{21}^2 a_0^3 \rho^4 \exp(-\rho)d\rho$$

$$= N_{21}^2 a_0^3 \int_0^1 \rho^4 \exp(-\rho)d\rho$$

$-\rho^4 e^{-\rho}$

$$= N_{21}^2 a_0^3 \left[-4\rho^3 e^{-\rho} - 12\rho^2 e^{-\rho} - 24\rho e^{-\rho} - 24e^{-\rho}\right]_0^1 = N_{21}^2 a_0^3\left[-1e^{-1} - 4\cdot 1e^{-1} - 12\cdot 1e^{-1} - 24e^{-1} - 24e^{-1} + 24e^{0}\right]$$

$$P = \frac{1}{24}\left[-65e^{-1} + 24\right] \approx \underline{0{,}0037}$$

d)

$$P = \int_0^{a_0} |R(r)|^2 r^2 dr = N_{20}^2 \int_0^{a_0} \left(2-\frac{r}{a_0}\right)^2 \exp\left(-r/2a_0\right)^2 r^2 dr = N_{20}^2 \int_0^{a_0} \left(2-\frac{r}{a_0}\right)^2 e^{(-r/a_0)} r^2 dr$$

$$= N_{20}^2 \int_0^{a_0} \left(4-\frac{4r}{a_0}+\frac{r^2}{a_0^2}\right) r^2 e^{(-r/a_0)} dr = N_{20}^2 \int_0^{a_0} \left(4r^2-\frac{4r^3}{a_0}+\frac{r^4}{a_0^2}\right) e^{(-r/a_0)} dr$$

$$= N_{20}^2 \int_0^1 \left(4\rho^2 a_0^2 - \frac{4\rho^3 a_0^3}{a_0} + \frac{\rho^4 a_0^4}{a_0^2}\right) e^{(-\rho a_0/a_0)} a_0 d\rho$$

$$= N_{20}^2 \int_0^1 a_0 \left(4\rho^2 a_0^2 - 4\rho^3 a_0^2 + \rho^4 a^2\right) e^{-\rho} d\rho = N_{20}^2 a_0^3 \int_0^1 \left(4\rho^2 - 4\rho^3 + \rho^4\right) e^{-\rho} d\rho$$

$$= \frac{1}{8a_0^3} a_0^3 \int_0^1 \left(4\rho^2 - 4\rho^3 + \rho^4\right) e^{-\rho} d\rho = \frac{1}{8}\left(\int_0^1 4\rho^2 e^{-\rho} d\rho - \int_0^1 4\rho^3 e^{-\rho} d\rho + \int_0^1 \rho^4 e^{-\rho} d\rho\right)$$

$$= \frac{1}{8}\left\{\left[-4\rho^2 e^{-\rho} - 8\rho e^{-\rho} - 8e^{-\rho}\right]_0^1 - \left[-4\rho^3 e^{-\rho} - 12\rho^2 e^{-\rho} - 24\rho e^{-\rho} - 24e^{-\rho}\right]_0^1\right.$$

$$\left. + \left[-\rho^4 e^{-\rho} - 4\rho^3 e^{-\rho} - 12\rho^2 e^{-\rho} - 24\rho e^{-\rho} - 24e^{-\rho}\right]_0^1\right\}$$

$$= \frac{1}{8}\left\{\left[-4e^{-1} - 8e^{-1} - 8e^{-1} + 8\right] - \left[-4e^{-1} - 12e^{-1} - 24e^{-1} - 24e^{-1} + 24\right] + \left[-e^{-1} - 4e^{-1} - 12e^{-1} + 24e^{-1} - 24e^{-1} + 24\right]\right\}$$

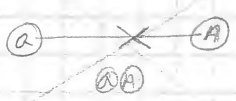
$$= \frac{1}{8}\left\{\left[-\frac{4}{e} - \frac{8}{e} - \frac{8}{e} + 8 + \frac{4}{e} + \frac{12}{e} + \frac{24}{e} + \frac{24}{e} + 24 - \frac{1}{e} - \frac{4}{e} - \frac{12}{e} + \frac{24}{e} - \frac{24}{e} + 24\right]\right\}$$

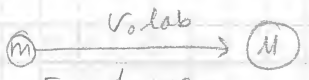
$$= \frac{1}{8}\left\{\left[-\frac{4}{e} - \frac{8}{e} - \frac{8}{e} + 8 - \frac{1}{e}\right]\right\} = \frac{1}{8}\left[-\frac{21}{e} + 8\right] = \underline{\underline{\frac{1}{8}\left(8-\frac{21}{e}\right) = 1{,}3 \cdot 10^{-2}}}$$

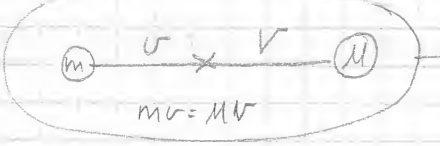
Bruker formelen: $\int x^n e^{ax} dx = \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} + \cdots + \frac{(-1)^n n!}{a^n}\right)$

4.7

a) $a + A \longrightarrow B + b$







$u - V = 0$

Da har vi følgende relasjoner:

$$v = \frac{M}{m} V \quad , \quad V = \frac{m}{M} v = u \qquad (1)$$

$$V_0 = v + u = v + V = v + \frac{m}{M} v = v\left(1 + \frac{m}{M}\right)$$

slik at (1) $$V = \frac{m}{M} v = \frac{m}{M} \frac{V_0}{\left(1+\frac{m}{M}\right)} = \frac{m}{M} \frac{V_0 M}{(M+m)} = \left(\frac{m}{M+m}\right) V_0$$

$$E_k^{cms} = \frac{1}{2} m v^2 + \frac{1}{2} M V^2 = \frac{1}{2} m v_0^2 \left(\frac{M}{(M+m)^2} + \frac{m}{(M+m)^2} \right) M = \frac{1}{2} m v_0^2 \left(\frac{1}{(M+m)} \right) M$$

$$= E_{kin}^{lab} \left(\frac{M}{M+m} \right)$$

b) Den formelle definisjonen for Q-verdien for reaksjonen er

$$a + A \longrightarrow B + b$$

$$\Rightarrow \quad Q: \quad m_a c^2 + M_a c^2 - M_B c^2 - m_b c^2 = 0$$

c)

$$E_{tot}^{cm} = E_{kin}^{cm} + Q = E_{kin}^{lab} \frac{M}{M+m} + Q$$

d) $$E_{kin}^{cm} \geq Q$$

4.8

a) ^4_2He (2) $\xrightarrow{E_k = 5{,}3\,MeV}$ ^9_4Be | (X) + γ

Bevaringslover:

B	4 + 9	13 + 0
Q	2 + 4	6

$^{13}_{6}X = {}^{13}_{6}C$

b)

$$m_2 c^2 + M_B c^2 - M_c c^2 + m_\gamma c^2 = 0$$

$$= (4{,}002603 + 9{,}012182 - 13{,}003353)\,u c^2 \cdot \frac{931{,}1\,MeV}{c^2} = 10{,}65 \cdot MeV$$

c) Energi:

$$K_\alpha + M_{Be}c^2 + M_{He}c^2 = m_c c^2 + K_c + K_\gamma$$

$K_\alpha = 5{,}3\,MeV$

$$K_\alpha + Q = K_c + K_\gamma = 15{,}95\,MeV \qquad (1)$$

Bevegelsesmengde:

$$P_\alpha = P_\gamma + P_c \Rightarrow P_\alpha \cdot c = P_\gamma \cdot c + P_c \cdot c = K_\gamma + P_c \cdot c$$

$$\sqrt{2m_\alpha K_\alpha c^2} = K_\gamma + \sqrt{2m_c K_c \cdot c^2}$$

$$\sqrt{2m_\alpha c^2 K_\gamma} = \sqrt{2 \cdot 4{,}002603 \cdot 931{,}5\,MeV \cdot 5{,}3\,MeV} = \underline{198{,}80\,MeV}$$

$$\sqrt{2m_c c^2} = 155{,}6\sqrt{MeV}$$

altså:

$$198{,}8\,MeV = K_\gamma + 155{,}6\sqrt{MeV \cdot K_c} \qquad (2)$$

$$K_c + K_\gamma = 15{,}95$$

$$198{,}8 = K_\gamma + 155{,}6\sqrt{K_c}, \quad x = \sqrt{K_c}$$

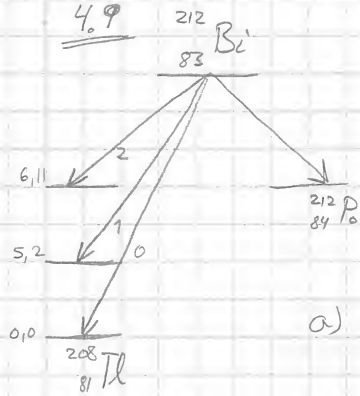
$$\left.\begin{aligned} x^2 + K_\gamma &= 15{,}95 \\ 198{,}8 &= K_\gamma + 155{,}6x \end{aligned}\right\} \quad x^2 - 155{,}6x + 182{,}2 = 0$$

$$\underline{x = 1{,}18}$$

$$K_c = x^2 = \underline{1{,}39\,MeV}$$

$$K_\gamma = 15{,}95\,MeV - 1{,}39\,MeV = \underline{14{,}56\,MeV}$$

4.9



Bevaringslover:

Bevegelsesmengde
Energi
ladning
Baryontallet (antall nukleoner)
Leptontallet

a) $^{212}_{83}Bi \longrightarrow {}^{212}_{84}Po + e^- + \bar{\nu}_e$ (β-decay)

$^{212}_{83}Bi \longrightarrow (^{208}_{81}Tl)^* + {}^4_2He$ (α-decay)

$(^{208}_{81}Tl)^* \rightarrow {}^{208}_{81}Tl + \gamma$ (γ-decay)

b) α-strålingen inneholder α partikler med 3 forskjellige energier bestemt av sluttilstanden i $^{208}_{81}Tl$.

$$E_{\alpha 0} = \left(M(^{212}_{83}Bi) - M(^{208}_{81}Tl) - M(^{4}_{2}He)\right)c^2 = 0{,}006665\,u = \underline{6{,}2085\,MeV}$$

$$E_{\alpha 1} = E_{\alpha 0} - 5{,}2\,keV = \underline{6{,}2033\,MeV}$$

$$E_{\alpha 2} = E_{\alpha 0} - 6{,}1\,keV = \underline{6{,}2024\,MeV}$$

γ-strålingen inneholder 3 forskjellige energier

$$E_{\gamma 1} = (6{,}1 - 5{,}2)\,keV = \underline{0{,}9\,keV}$$

$$E_{\gamma 2} = (6{,}1 - 0{,}0)\,keV = \underline{6{,}1\,keV}$$

$$E_{\gamma 3} = (5{,}2 - 0{,}0)\,keV = \underline{5{,}2\,keV}$$

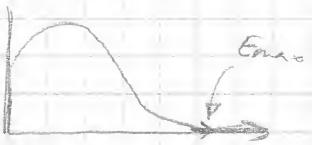
β-decay:

$$Q = (m_{Bi} - m_{Po} - m_e)c^2$$

$$= [(M_{Bi} - 83m_e) - (M_{Po} - 84m_e) - m_e]c^2 = (M_{Bi} - M_{Po})c^2$$

$$= (211{,}9912552 - 211{,}988842) \cdot 931{,}5\,MeV$$

$$= \underline{2{,}25\,MeV} = E_{kmax}$$



c) $^{212}Bi \longrightarrow {}^{212}Po + e^- + \bar{\nu}$

L: 0 — 0 + 1 − 1

B: 212 — 212 + 0 + 0

4-10

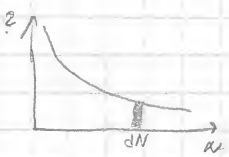
a) $$-\frac{dN(t)}{dt} = \lambda N(t) \qquad \int \frac{dN(t)}{dt} = -\lambda \int dt$$

$$\ln N(t) = \ln C - \lambda t \quad \Longleftrightarrow \quad \underline{N(t) = N_0 e^{-\lambda t}}$$

$$N(t) = Ce^{-\lambda t}, \text{ ved } t=0 \text{ så er } C = N_0(t=0)$$

b)

$$N_0 \bar{t} = \int_{N_0}^{0} t\,dN(t) = -\int_0^{\infty} t\,\lambda N(t)\,dt = \frac{N_0}{\lambda}$$



$$N(t) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \Rightarrow +\ln 2 = +\lambda t_{1/2}$$

$$t_{1/2} = \frac{0{,}693}{\lambda} = 0{,}693\,\bar{\tau}$$

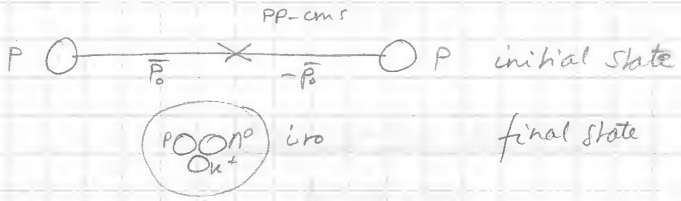
c)

	$P+P \longrightarrow P+\Lambda^0+X$
B	$1+1 \longrightarrow 1+1+0$
Q	$1+1 \longrightarrow 1+0+1$
S	$0+0 \longrightarrow 0-1+1$

X har B=0, Q=1, S=+1 altså en meson

$X = K^+(u\bar{s})$

d)



$pp \longrightarrow pn^0K^+$

initial state

final state

Dette er invariant: $m^2c^4 = E^2 - p^2c^2$

$$M^2c^4 = (m_p + m_n + m_K)^2c^4 = E^2 - p^2c^2$$

$$(m_p + m_\Lambda + m_K)^2c^4 = \sqrt{m_{p_1}^2c^4 + \bar{p}_0^2c^2} + \sqrt{m_{p_2}^2c^4 + c^2\bar{p}_0^2} - (+\bar{p}_0 - \bar{p}_0)^2$$

$$= (\Sigma E_i)^2 - (\Sigma p_i c)^2 \qquad \vec{P} = +\bar{p}_0 - \bar{p}_0 = 0$$

$$(m_p + m_\Lambda + m_K)^2c^4 = (\Sigma E_i)^2 - (\Sigma p_i c)^2$$

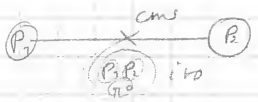
$$= \left(\sqrt{m_{p_1}^2c^4 + p_0^2c^2} + m_{p_2}c^2\right)^2 - \left(\vec{p}_0 c - 0 \cdot c\right)^2$$

det eneste ukjent er p_0 finn den, så har vi svaret.

$$\sqrt{(m_p + m_\Lambda + m_K)^2c^4 + \bar{p}_0^2c^2} = \sqrt{m_{p_1}^2c^4 + \bar{p}_0^2c^2} + m_{p_2}c^2$$

4.13

$p_1 + p_2 \longrightarrow p_1 + p_2 + \pi^0$



$\vec{p}_i + \vec{0} = ?$

$$(\Sigma E_i)^2 - (\Sigma p_i c)^2 = (M_p + M_p + M_{\pi^0})^2c^4$$

$$\left(\sqrt{m^2c^4 + p_0^2c^4} + m_p c^2\right)^2 - \left(\vec{p}_0 c + 0 \cdot c\right)^2 = (M_p + M_p + M_{\pi^0})^2c^4$$

Vi har følgende kvantetall: baryontall B
ladning Q
leptontall L_e, L_μ, L_τ
strangeness S

4.14

	$p+p \longrightarrow p+\gamma$
B	$1+1 \neq 1+0$
L	$0+0 = 0+0$
Q	$1+1 \neq 1+0$

	$p+p \longrightarrow p+p+e^-$
B	$1+1 = 1+1+0$
L_e	$0+0 \neq 1+1-1$
Q	$0+0 = 0+0+1$

	$p+p \longrightarrow \pi^0+p+p$ (ok)
B	$1+1 = 0+1+1$
L	$0+0 = 0+0+0$
Q	$1+1 = 0+1+1$ → mulighetet

	$p+p \longrightarrow p+p+k^+$
B	$1+1 = 1+1+0$
Q	$1+1 \neq 1+1+1$
S	$0+0 \neq 0+0+1$

	$p+p \longrightarrow p+p+k^+ +k^0$
B	$1+1 = 1+1+0+0$
Q	$1+1 = 1+1+1-1$ ← 0?
S	$0+0 \neq 0+0+1+1$

mesoner er ustabile

	$p+p \longrightarrow e^-+e^+$
B	$1+1 \neq 0+0$
Q	$1+1 = 1-1$
S	$1+1 = -1+1$
L	$0+0 = +1-1$

	$p+p \longrightarrow e^+ + e^- + \mu^+$
B	$1+1 \neq 0+0+0$
Q	$1+1 \neq -1+1+1$
L_e	$0+0 = +1-1+0$
L_μ	$0+0 \neq 0+0-1$
S	$0+0 = 0+0+0$

	$p+p \longrightarrow e^+ + e^- + \nu_e$
B	$1+1 \neq 0+0+0$
Q	$1+1 \neq +1-1+0$
L_e	$0+0 = -1+1+1$

13.5.94 KORSMO

e)

$p + p \longrightarrow p + \Lambda + K^+$

$$m_0c^2 + \sqrt{m_p^2c^4 + p_0^2c^2} = \sqrt{(m_p + m_\Lambda + m_K)^2c^4 + p_0^2c^2}$$

$$m_p^2c^4 + \sqrt{m_p^2c^4 + p_0^2c^2} - p_0^2c^2 = (m_p + m_\Lambda + m_K)^2c^4$$

invariant masse:

$$E^2 = p^2c^2 + m_0^2c^4 \Rightarrow m_0^2c^4 = E^2 - p^2c^2$$ invariant kommer vert

[relativistisk kinematikk]

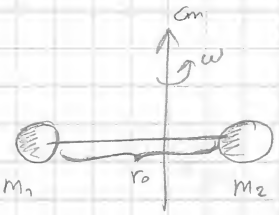
$$\left(\sum E\right)^2 - \left(\sum \vec{p}\right)c^2 = M_0^2c^4 \quad , M_0 \text{ - invariant masse}$$

$$(m_pc^2 + E_p)^2 = (m_p + m_\Lambda + m_K)^2c^4$$

$$E_p = \frac{(m_p + m_\Lambda + m_K)^2 - 2m_p^2}{2m_p}c^2$$

$$p_0 = \frac{1}{c}\sqrt{E_p^2 - m_p^2c^4}$$

4.11



$$I = \mu r_0^2 \qquad \mu = \frac{M_1M_2}{M_1 + M_2}$$

$$E_{rot} = \frac{L^2}{2I}$$ egenfunksjonene

$$L^2 = l(l+1)\hbar^2 \qquad l = 0, 1, 2, 3$$

$$E_l = \frac{l(l+1)\hbar^2}{2I}$$ egenverdiene

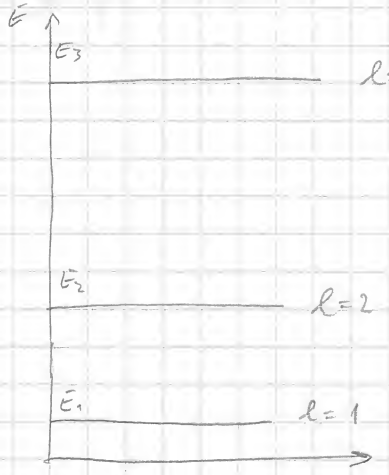
$l = 0$, $E_0 = 0$

$l = 1$ $E_1 = \frac{2\hbar^2}{2I} = \frac{\hbar^2}{I}$

$l = 2$ $E_2 = \frac{6\hbar^2}{2I} = \frac{3\hbar^2}{I}$

$l = 3$ $E_3 = \frac{6\hbar^2}{I}$

$l = 4$ $E_4 = \frac{10\hbar^2}{I}$



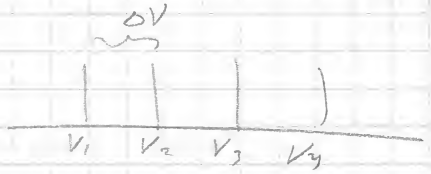
$$E_l = \frac{l(l+1)\hbar^2}{2I}$$

$$h\nu_l = \Delta E = E_l - E_{l-1} = \frac{l(l+1)\hbar^2}{2I} - \frac{(l-1)l\hbar^2}{2I}$$

$$= \frac{\hbar^2}{2I}(l^2+l-l^2+l) = \frac{\hbar^2}{I} l$$

$$h\nu_{l+1} - h\nu_l = \frac{\hbar^2}{I}(l+1) - \frac{\hbar^2}{I} l = \underline{\underline{\frac{\hbar^2}{I}}}$$

$$\Delta\nu = \frac{\hbar^2}{hI} = \frac{\hbar^2}{(2\pi)^2 \hbar I} = \underline{\underline{\frac{\hbar}{4\pi^2 I}}} \quad \Rightarrow$$



c)

HCl

$$\Delta E = 2{,}02 \cdot 10^{-3}\,eV = \frac{\hbar^2}{I} \quad \Leftrightarrow \quad I = \frac{\hbar^2}{\Delta E} = \frac{(6{,}58 \cdot 10^{-16}\,eV \cdot s)^2}{2{,}62 \cdot 10^{-3}\,eV} =$$

$$1{,}65 \cdot 10^{-28}\,eV \cdot s^2 = 1{,}65 \cdot 10^{-28}\,\frac{eV}{s^2} \cdot 1{,}602 \cdot 10^{-19}\,J/eV$$

$$= \underline{\underline{2{,}65 \cdot 10^{-47}\,kg/m^2}}$$

d) Hatom $m = 1u$ m_1

Cl-atom $m = 35u$ m_2

$$I = \mu r_0^2$$

$$\mu = \frac{M_1 \cdot M_2}{M_1 + M_2} = \frac{35u^2}{36u} = \frac{35}{36}u$$

$$r_0 = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{2{,}65 \cdot 10^{-47}\,kg\,m^2}{\frac{35}{36} \cdot 1{,}66 \cdot 10^{-27}\,kg}} = \underline{\underline{1{,}28 \cdot 10^{-8}\,m}}$$

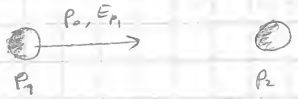
næste gang

2.10

3.12

4.13

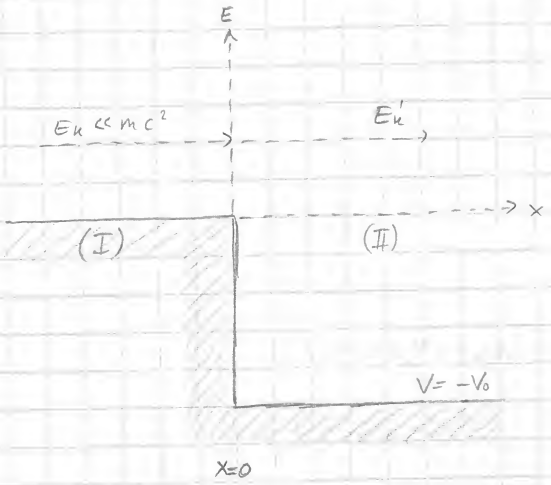
$$P_1 + P_2 \longrightarrow P_1 + P_2 + \pi^0$$



$$\left(m_{P_1}c^2 + \underbrace{\sqrt{m_p^2c^4 + p_0^2c^2}}_{E_{P_1}}\right)^2 - p_0^2c^2 = (m_p + m_p + m_\pi)^2c^4$$

$$E_{P_1} = \frac{(2m_p + m_\pi)^2 - 2m_p^2}{2m_p} \cdot c^2$$

$$\underline{\underline{p_0 = \frac{1}{c}\sqrt{E_{P_1}^2 - m_p^2c^4}}}$$



$V = 0$, $x < 0$

$V = -V_0$, $x > 0$

a)

$E_k = E - V$

$x < 0$: $\underline{\underline{E_k = E}}$

$x > 0$: $E_k' = E - (-V_0) = \underline{\underline{E + V_0}}$

b) $\psi_I = Ae^{ikx} + Be^{-ikx}$, $\psi_{II} = Ce^{ik_2 x} + De^{-ik_2 x}$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\psi(x) = E\,\psi(x)$$

$$\frac{d^2\psi}{dx^2} = \frac{-2m(E-V)}{\hbar^2}\,\psi \qquad \text{generelt}$$

I, $x < 0$:

$$\frac{d^2\psi}{dx^2}_I = \frac{-2mE}{\hbar^2}\psi = -k_1^2\psi_I \;, \quad k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Løsning: $\psi_I = \underset{\rightarrow}{Ae^{ik_1 x}} + \underset{\leftarrow}{Be^{-ik_1 x}}$

II, $x > 0$:

$$\frac{d^2\psi_{II}}{dx^2} = -\frac{2m(E+V_0)}{\hbar^2}\psi_{II} = -k_2^2\psi_{II} \;, \quad k_2 = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

Løsning: $\psi_{II} = \underset{\rightarrow}{Ce^{ik_2 x}} + \underset{\leftarrow}{De^{-ik_2 x}}$, $D = 0$ for det skyer ingen refleksjon

c) ψ, ψ' kontinuerlig ved $x = 0$

$x = 0$: $\psi_I = \psi_{II}$ $\qquad \psi_I' = \psi_{II}'$

$A + B = C$ $\qquad ik_1 A - ik_2 B = ik_2 C$

$$A + B = \frac{k_1}{k_2}(A - B) \Rightarrow 1 + \frac{B}{A} = \frac{k_1}{k_2}\left(1 - \frac{B}{A}\right)$$

$$\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2} \qquad \left|\frac{A}{B}\right|^2 = \frac{(k_1 - k_2)^2}{(k_2 + k_1)^2}$$

$$C - A = A - \frac{k_2}{k_1} C \Rightarrow \frac{C}{A} - 1 = 1 - \frac{k_2}{k_1}\frac{C}{A}$$

$$\frac{C}{A} = \frac{2k_1}{k_1+k_2} \Rightarrow \underline{\underline{\left|\frac{C}{A}\right|^2 = \frac{4k_1^2}{(k_1+k_2)^2}}}$$

d)

$$R = \left|\frac{B}{A}\right|^2 \quad , T = ?$$

$$R = \frac{\text{fluks (reflektert)}}{\text{fluks (innsendt)}} \qquad T = \frac{\text{fluks (transmittert)}}{\text{fluks (innsendt)}}$$

fluks - antall partikler som passerer / arealenhet, pr tid $= n_1 |A|^2 \cdot v$

altså:

$$R = \frac{n \cdot v |B|^2}{n \cdot v |A|^2} = \left|\frac{B}{A}\right|^2 = \frac{(k_1-k_2)^2}{(k_1+k_2)^2}$$

$$T = \frac{n \cdot v |C|^2}{n \cdot v |A|^2} = \underline{\underline{\frac{k_2}{k_1}\frac{|C|^2}{|A|^2}}} = \frac{k_2}{k_1}\frac{4k_1^2}{(k_1+k_2)^2} \qquad v = \frac{p}{m} = \frac{\hbar k}{m}$$

e) $$R+T = \frac{k_1^2 + k_2^2 - 2k_1k_2 + 4k_1k_2}{(k_1+k_2)^2} = \frac{k_1^2+k_2^2+2k_1k_2}{(k_1+k_2)^2} = \underline{\underline{1}}$$

3.12 (V93)

$$\hat{H}_0 \psi_{nlm_l} = -\frac{E_0}{n^2} \psi_{nlm_l} \qquad n = 1,2,3,\ldots$$

a)

$$\hat{H}_0 = \frac{-\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$n \geq 1$, $n = 1,2,3$

$0 \leq l \leq n-1$, $l = 0,1,2,\ldots,n-1$

$|m_l| \leq l$, $m_l = 0, \pm 1, \pm 2, \ldots, \pm l$

b)

$$\left(\frac{d^2}{d\varphi^2} + m_l^2\right)\Phi(\varphi) = 0 \quad \Rightarrow \quad \frac{d^2\Phi}{d\varphi^2} = -m_l^2\Phi$$

$\Phi(\varphi) = Ae^{im_l\varphi} + Be^{im_l\varphi}$ (egenvardi for $\hat{L}_z$)

$\Phi(\varphi) = \Phi(\varphi + 2\pi)$

$e^{im_l\varphi} = e^{im_l\varphi} e^{im_l 2\pi} = 1$, $m_l = 0, \pm 1, \pm 2, \pm 3, \ldots$, $\underline{e^{2\pi i} = 1}$

$e^{i\theta} = \cos\theta + i\sin\theta$

c)

$$\psi_{200} = \sqrt{\frac{1}{8\pi a_0^3}}\left(1 - \frac{\rho}{2}\right)e^{-\rho/2} \quad , \rho = \frac{r}{a_0} \qquad \int_0^\infty \rho^k e^{-\rho} d\rho = k!$$

$$\int \psi_{200}^* \psi_{200}\, dV = \frac{1}{8\pi a_0^3}\int_0^\infty\int_0^{2\pi}\int_0^\pi \left(1 - \frac{\rho}{r}\right)^2 e^{-\rho} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$= \frac{1}{8\pi a_0^3} 2\cdot 2\pi \int_0^\infty \left(1 - \frac{\rho^2}{4} - \rho\right) e^{-\rho} (a_0\rho)^2 a_0 \, d\rho = \frac{1}{2}\int \left(\rho^2 + \frac{\rho^4}{4} - \rho^3\right) e^{-\rho} d\rho$$

$$= \frac{1}{2}\left(2! + \frac{1}{4}4! - 3!\right)$$

$$= \frac{1}{2}(2 + 6 - 6) = \underline{1}$$

$$\langle r \rangle = \int \psi_{200}^* \, r \, \psi_{200} \, dV = \frac{1}{8\pi a_0^3} 2\pi \cdot 2 \, a_0^4 \int_0^\infty \rho^3\left(1 + \rho^2/4 - \rho\right) e^{-\rho} d\rho$$

$$= \frac{1}{2}a_0\left(3! + \frac{1}{4}5! - 4!\right) = \frac{1}{2}a_0(6 + 30 - 24) = \underline{6a_0}$$

$P(r) = r^2 \psi^* \psi$ $\rightarrow \iiint \rho \, dV = \iiint |\psi(r,\theta,\phi)|^2 r^2 \sin\theta \, d\theta \, d\phi \, dr$

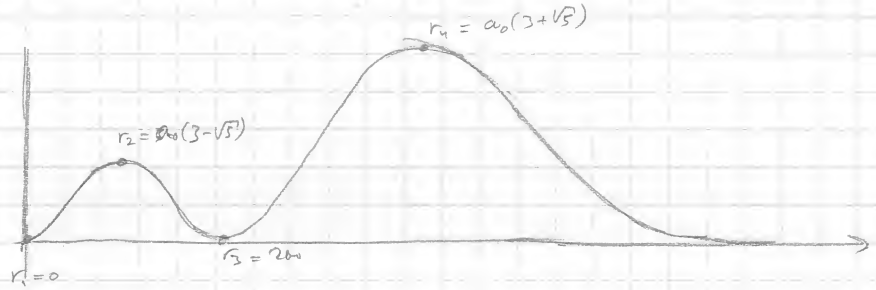
$$= \int_{r_1}^{r_2} \underbrace{4\pi r^2 |\psi(r,\theta,\phi)|^2}_{P(r)} dr$$

$$= r^2 \frac{1}{8\pi a_0^3} (1 - s/2)^2 e^{-s} = r^2 \left(1 - \frac{r}{2a_0}\right) e^{-r/a_0}$$

$$\frac{dP}{dr} = r \left(1 - r/2a_0\right) \left(2 + \frac{3r}{a_0} + \frac{r^2}{2a_0^2}\right) = 0$$

$\Rightarrow r = 0 \quad r = 2a_0$

$r = a_0 (3 \pm \sqrt{5})$

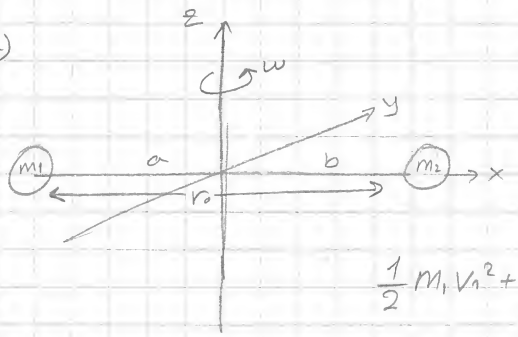


d) $\hat{H}_1 = \frac{E_0}{16\hbar^2} (\hat{L}_x^2 + \hat{L}_y^2) \quad , \hat{H} = \hat{H}_0 + \hat{H}_1$

$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$

4.11

a)



$m_1 a = m_2 b$

$a + b = r_0$

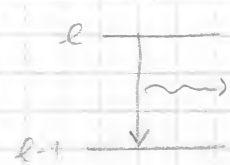
$a = r_0 \frac{m_2}{m_1+m_2}$ $\qquad b = r_0 \frac{m_1}{m_1+m_2}$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \Rightarrow \frac{1}{2} m_1 (a\omega)^2 + \frac{1}{2} m_2 (b\omega)^2 = \frac{L^2}{2I}, \quad I = m_1 a^2 + m_2 b^2$$

$$H = \frac{\hat{L}^2}{2I} \Leftrightarrow \hat{H} = \frac{\hat{L}^2}{2I}$$

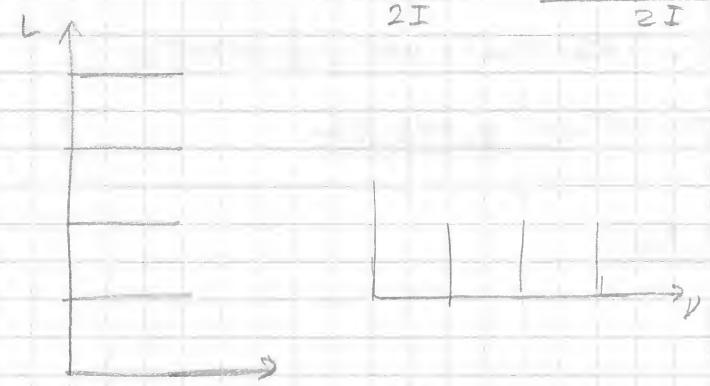
$$\hat{H}\Psi = \frac{\hat{L}^2}{2I}\Psi = \frac{l(l+1)\hbar^2}{2I}\Psi = E(l)\,\Psi$$

b)



Linjene har konstant avstand.

$$\frac{l(l+1)\hbar}{2I} - \frac{(l-1)(l-1+1)\hbar}{2I} = \frac{2l}{2I}\hbar^2 = \frac{l}{I}\hbar^2$$



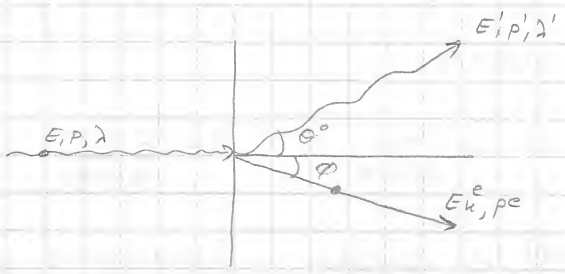
c) $\Delta E = 2{,}62 \cdot 10^{-3}\,eV$ $\qquad E_1 - E_0 = \Delta E = \frac{1 \cdot \hbar^2}{I} = 2{,}62 \cdot 10^{-3}\,eV$

$$I = 2{,}65 \cdot 10^{-47}\,kgm^2$$

d)

$I = \mu r_0^2 \Rightarrow r_0 = 7{,}26 \cdot 10^{-10}\,m$

1.7 Compton-spredning



Vi har følgende informasjon:

$$\lambda = 1{,}00 \cdot 10^{-11}\,m = 0{,}01\,nm$$

$$\theta = 60°$$

$$\Delta\lambda = \lambda' - \lambda = \lambda_c (1 - \cos\theta)$$

$$\lambda_c = 2{,}426 \cdot 10^{-12}\,m$$

a) $E = h\nu = \frac{hc}{\lambda} = \frac{1240\,eV\,nm}{0{,}01\,nm} = \underline{1{,}24 \cdot 10^5\,eV}$

$$E = pc \leftrightarrow p = \frac{E}{c} = \frac{hc}{c\lambda} = \frac{h}{\lambda} = \underline{1{,}24 \cdot 10^5\,eV/c}$$

b) Comptons formel:

$$\Delta\lambda = \lambda_c (1 - \cos\theta) = 2{,}426 \cdot 10^{-12}\,m\,(1 - \cos\theta) = 0{,}1213 \cdot 10^{-11}\,m$$

$$\lambda' = \lambda + \Delta\lambda = 1{,}1213 \cdot 10^{-11}\,m = \underline{0{,}011213\,nm}$$

$$E' = \frac{hc}{\lambda'} = \frac{1240\,eV\,nm}{0{,}011213\,nm} = \underline{1{,}11 \cdot 10^5\,eV}$$

$$p' = E'/c = \underline{1{,}11 \cdot 10^5\,eV/c}$$

c)

$$E_\lambda = E_\lambda' + E_e' \longrightarrow E_e' = E_\lambda - E_\lambda' = 1{,}24 \cdot 10^5\,eV/c - 1{,}11 \cdot 10^5\,eV/c$$

$$\vec{P}_\lambda = \vec{P}_\lambda' + \vec{P}_e' \qquad = \underline{0{,}13 \cdot 10^5\,eV}$$

$$P_e' = \sqrt{2 m_e E_e'} = \sqrt{2 \cdot 0{,}511 \cdot 10^6\,eV/c^2 \cdot 0{,}13 \cdot 10^5\,eV} = \underline{1{,}15 \cdot 10^5\,eV/c}$$

x: $P_\lambda = P_\lambda' \cos\theta + P_e' \cos\varphi$

y: $0 = P_\lambda' \sin\theta + P_e' \sin\varphi \qquad \sin\varphi = \frac{P_\lambda'}{P_e'} \sin\theta = -\frac{1{,}1}{1{,}15} \sin 60° \Rightarrow \varphi = \underline{-56{,}7°}$